

**Ministry of Education and Science of Ukraine  
Lutsk National Technical University**



**PHYSICS**  
**COLLECTION OF PROBLEMS**

**for students of the first bachelor's degree of higher education of  
full and external forms education**

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Signed to the publication

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**Physics. Collection of problems:** Study guide for undergraduate students of

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The study guide provides examples of solving problems in physics and problems for  
self-solving on topics covering all sections of the physics course. A brief list of formulas and  
laws relating to the performance of tasks on the relevant topic is presented at the beginning of  
each section.

The study guide is intended for students of engineering and technical specialties of  
full and external forms education of LNTU.

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## **INTRODUCTION**

The ability to solve problems is one of the main criteria for mastering the discipline of “physics”. Mastering special methods and techniques, which are given in problem solving examples, contributes to the success of the tasks.

The collection of problems is grouped by the main sections of the general course of physics in accordance with the current program.

Such tasks are one of the forms of testing the acquisition of theoretical knowledge of the course of physics in the education system as well as the acquisition of skills of their practical use in solving problems.

The formed problems are divided into separate sections. A short list of forms and laws is given at the beginning of each. They relate to the tasks on the relevant topic. These formulas allow the student to form an idea of the amount of theoretical material to be studied. Formulas can serve as a formal apparatus for solving problems. Reference tables and list of recommended reading are given at the end of the textbook.

The proposed tasks can be used for independent work by students and to control the knowledge of students by the teacher.

## **PROGRAM OF EDUCATIONAL DISCIPLINE “PHYSICS”**

### **Mechanics**

Mechanical motion as the simplest form of motion of matter. Frame of reference. Kinematic equations of motion of a material point. Trajectory, path length, displacement vector. Speed. Acceleration and its components.

Newton’s first law. Inertial reference systems. Mass. Force. Newton’s second law. Impulse of a material point. Newton’s third law. The law of conservation of momentum and its relation to the homogeneity of space. The center of mass of the mechanical system and the law of its movement.

Energy as a universal measure of different forms of movement and interaction. The work of the force and its expression through the curvilinear integral. Power. The kinetic energy of the mechanical system and its connection to the work of external forces applied to the system. Conservative forces. Potential field of forces. The potential energy of a material point in an external force field and its connection with the force acting on a material point. The concept of the gradient of a scalar function. The law of conservation of mechanical energy and its connection with the homogeneity of space. The law of conservation and transformation of energy as a manifestation of the indestructibility of matter and its movement. Mechanical shock. Strike absolutely elastic and inelastic bodies.

Elements of kinematics of rotational motion. Angular velocity and angular acceleration and their connection with linear velocity and acceleration. The moment of inertia of the body relative to the axis. Steiner’s theorem. The kinetic energy of rotation. Moment of strength. Work with rotational motion. The main equation of dynamics of rotational motion of a solid body. The momentum of the

body relative to the fixed axis of rotation. The law of conservation of momentum and its relation to the isotropy of space.

Law of universal gravitation. Gravity and weight. Weightlessness. The field as a form of matter that performs force interaction between particles of matter. Gravity field and its tension. Work in the field of gravity. The potential of the field.

### **Molecular Physics and Thermodynamics**

Statistical and thermodynamic research method. Thermodynamic parameters. Equilibrium states and processes.

The basic laws of ideal gas. Equation of state of an ideal gas. The basic equation of the molecular-kinetic theory of ideal gases. The average kinetic energy of molecules. Molecular-kinetic interpretation of absolute temperature.

Maxwell's law for the distribution of ideal gas molecules by velocities. Barometric formula. Boltzmann's law for the distribution of particles in an external potential field. The average number of collisions and the average free path of molecules. Phenomena of transfer in thermodynamic non-equilibrium systems. Molecular-kinetic theory of these phenomena. Diffusion, Fick's law. Internal friction, Newton's law. Thermal conductivity, Fourier's law.

The internal energy of an ideal gas. The number of degrees of free molecules. The law of uniform distribution of energy by degrees of free molecules. Work and heat are two forms of energy transfer to a thermodynamic system. The first principle of thermodynamics. Gas operation when changing its volume. Heat capacity. Application of the first principle of thermodynamics to isoprocesses. Adiabatic process. Poisson's equation. Work in the adiabatic process. Reversible and irreversible processes. Circular process. Entropy, its statistical content and relation to thermodynamic probability. The second principle of thermodynamics and its statistical content. Carnot cycle and its Efficiency for an ideal gas. Carnot's theorem.

Real gases. Van der Waals equation. Van der Waals isotherms and their analysis. Critical state of the substance. Critical state of substances. Phase transitions of the first and second kind. Internal energy of real gas. Crystal bodies and their features. Classification of crystals. Defects in crystals.

### **Electrostatics**

Electric charge. The law of conservation of electric charge. Conductors, dielectrics and semiconductors. Coulomb's law.

Electric field. Electric field strength. Electric field lines. The principle of superposition of electrostatic fields. Electric field intensity vector flow. The Ostrogradsky-Gauss theorem for electrostatic field in vacuum. Application of the Ostrogradsky-Gauss theorem for the calculation of electrostatic fields in vacuum. Field operation when moving electric charge in an electrostatic field. Circulation of the tension vector. Potential of the electrostatic field. The

relationship between potential and tension of the electrostatic field. Equipotential surfaces.

Electrical capacity of a separate conductor. Capacitors. Capacitor capacity. Connections of capacitors. The energy of a separate conductor, charged capacitor. The energy of the electrostatic field. Volumetric energy density.

Types of dielectrics. Polarization of dielectrics. Types of polarization. Polarization. Field strength in the dielectric. Free and bound charges in dielectrics. Dielectric permeability of the medium.

Electric displacement vector. Ostrogradsky-Gauss theorem for an electrostatic field in a dielectric.

The concept of ferroelectrics. Piezoelectric effect. Piezoelectrics.

### **Direct electric current**

Direct electric current and conditions of its existence. Strength and density of current. Extraneous forces. Electromotive force. Voltage. Ohm's law for a section of a circle. Resistance of conductors. Operation and power of current. Joule-Lenz law. Ohm's law for an inhomogeneous section of a circle (generalized Ohm's law). The work of the output of electrons from the metal.

The work of the output of electrons from the metal. Thermoelectronic emission. Current in gases. Ionization energy. Non-independent gas discharge. Independent gas discharge and its types (smoldering, spark, arc crown).

### **Electromagnetism**

Magnetic current field. Induction and tension of the magnetic field. Ampere's Law. Bio-Savar-Laplace law and its application to the calculation of the magnetic field of rectilinear and circular currents. Lorentz force. The motion of charged particles in a magnetic field. Hall effect. Circulation of magnetic induction vector. The law of full current for the magnetic field in a vacuum. Vortex character of the magnetic field. Magnetic flux. Ostrogradsky-Gauss theorem for a magnetic field in a vacuum. Conductor operation when moving and contour with current in a magnetic field.

The phenomenon of electromagnetic induction. Faraday's law of electromagnetic induction. Lenz's rule. Derivation of Faraday's law from the law of energy conservation. Application of the phenomenon of electromagnetic induction. Generators. Phenomenon of self-induction. Inductance of the contour. Current when closing and opening the circle. Mutual induction. The energy of the magnetic field. Volume energy density of the magnetic field.

Magnetics. Magnetic moments of electrons and atoms. Elementary theory of dia- and paramagnetism. Magnetic field in matter. Magnetization. Magnetic susceptibility and insight of the substance. The law of full current for magnetic field in matter. Ferromagnetics and their properties. The magnetization curve. Magnetic hysteresis. Curie point. The nature of ferromagnetism.

Vortex electric field. Maxwell's first equation. Displacement current. Generalized theorem on the circulation of the magnetic field intensity vector

(Maxwell's second equation). Maxwell's equation in integral and differential forms for the electromagnetic field.

### **Mechanical and electromagnetic oscillations and waves**

Harmonic oscillations and their characteristics. Differential equation of harmonic oscillations. Mechanical harmonic oscillations. Speed and acceleration with harmonic oscillating motion. Energy of harmonic oscillations. Harmonic oscillator. Elastic, mathematical and physical pendulums. Free harmonic oscillations in the oscillatory contour. Thomson's formula. Addition of harmonic oscillations of the same direction and frequency. Beatings. Addition of mutually perpendicular oscillations. Figures of the Lissage.

Fading mechanical and electromagnetic oscillations. Differential equation of fading oscillations and its solution. Logarithmic decrement of attenuation. Aperiodic process. Self-oscillation. Forced mechanical and electromagnetic oscillations. Differential equation of forced oscillations and its solution. Amplitude and phase of forced oscillations. Resonance. Propagation of waves in an elastic medium. Longitudinal and transverse waves. Wavelength. An equation of a running wave. A wave number. Phase velocity and wave dispersion. Wave equation. The principle of superposition of waves. Group speed. Wave interference. Coherent waves. Standing waves. The equation of the running wave. Wave number. Phase velocity and wave dispersion. Wave equation. The principle of superposition of waves. Group speed. Interference of waves. Coherent waves. Standing waves. The equation of the standing wave. Nodes and antinodes of a standing wave.

Radiation of electromagnetic waves. Scale of electromagnetic waves. Differential equation of electromagnetic wave. The main properties of electromagnetic waves. The speed of propagation of an electromagnetic wave. The energy of electromagnetic waves. The flow of energy. Poynting vector (Umov-Poynting vector). Dipole radiation. Application of electromagnetic waves.

### **Optics**

Development of the teaching of light. The basic laws of geometric optics. Full reflection.

Coherency and monochromaticity of light waves. Time and length of coherence. Interference of light. Methods of observing the interference of light. Calculation of the interference picture from two coherent sources. Optical path length. Interference of light in thin films. Stripes of equal inclination and equal thickness. Application of interference. Interferometers.

Diffraction of light. Huygens-Fresnel principle. Fresnel zone method. Rectilinear propagation of light. Fresnel diffraction on a round hole and disc. Fraunhofer diffraction on one slit and lattice. Diffraction on the spatial lattice. Wulf-Bragg formula. X-ray diffraction. Investigation of crystal structure.

Light dispersion. Normal and abnormal dispersion. Electronic theory of light dispersion. Light scattering. Rayleigh's Law. Absorption of light. Bouguer's

law. Vavilov-Cherenkov radiation. Natural and polarized light. Malus's law. Polarization when reflecting and refracting light. Brewster's law. Double refraction. Polarizing prisms and polaroids. Artificial optical anisotropy. Rotation of the polarization plane.

Thermal radiation and its characteristics. Absolutely black body. Kirchhoff's Law. Distribution of energy in the spectrum of an absolutely black body. Stefan-Boltzmann's laws and Wien's displacement. Quantum hypothesis and Planck's formula. Optical pyrometry. Types of photovoltaic effect and its laws. Einstein's equation for an external photoelectric effect. Photons. Application of photo effect. Mass and momentum of photon. Light pressure. Quantum and wave explanation of light pressure. Compton effect. Dialectical unity of corpuscular and wave properties of electromagnetic radiation.

### **Atomic and nuclear physics**

Elements of the zone theory of solids. Distribution of conduction electrons in metal by energy. Fermi Energy. Energy zones in crystals. Distribution of electrons in energy zones. Valence zone and conduction zone. Metals, dielectrics, semiconductors. Fermi level. Own and impurity conductivity of semiconductors. p-n-transition and its volt-ampere characteristic. Photoconductivity of semiconductors. Luminescence of solids.

Rutherford's nuclear model of the atom. The linear spectrum of the hydrogen atom. Serial terms. Bohr's theory for hydrogen-like systems. Franck-Hertz experiment.

Corpuscular-wave dualism properties of microparticles. The formula of De Broglia. Properties of de Broglie waves. The Heisenberg uncertainty ratio as a manifestation of corpuscular-wave dualism. Hydrogen-like system in quantum mechanics. Main, orbital and magnetic quantum numbers. Spatial quantization. Stern and Gerlach's experiment. Electron spin, spin quantum number.

Pauli principle. Distribution of electrons in an atom by states. X-rays. Brake and characteristic X-rays. Absorption, spontaneous and forced radiation. Optical quantum generators.

Charge, size and mass of the atomic nucleus. Mass and charge numbers. Core composition. Nucleons. Mass defect and binding energy. The moment of pulse of the core and its magnetic moment. The concept of the properties and nature of nuclear forces. Core models. Natural and artificial radioactivity. Law of radioactive decay. Nuclear reactions and their main types. Nuclear fission reaction. Separation chain reaction. Reaction of atomic nuclei synthesis. Elementary particles, their classification and mutual transformability.

## SECTION 1. MECHANICS

### Basic laws and formulas

Average speed	$v_{cp} = \frac{\Delta S}{\Delta t}$
Instant speed	$v = \frac{dS}{dt}$
Average acceleration of motion	$a_{cp} = \frac{\Delta v}{\Delta t}$
Instant acceleration	$a = \frac{dv}{dt}$
Equation of uniformly accelerated motion	$S = S_0 + v_0 t + \frac{at^2}{2}$
Speed of uniformly accelerated motion	$v = v_0 + at$
Tangential acceleration	$a_\tau = \frac{dv}{dt}$
Normal acceleration	$a_n = \frac{v^2}{R}$
Full acceleration	$a = \sqrt{a_\tau^2 + a_n^2}$
Impulse of the body	$\vec{p} = m\vec{v}$
Newton's Second Law	$a = \frac{F}{m} ; \quad \vec{F} = \frac{d(m\vec{v})}{dt}$
Law of Universal Attraction	$F = \gamma \frac{m_1 m_2}{R^2}$
Force of elasticity	$F = -kx$
Kinetic energy of gradual motion	$W_k = \frac{mv^2}{2}$
Potential energy of the body raised to a height $h$	$W_n = mgh$
Potential energy of compressed spring	$W_n = \frac{kx^2}{2}$
Work of constant force	$A = FS \cos \alpha$
Work of non-constant force	$A = \int_a^b F \cos \alpha dS$

Work as a change in body energy	$A = W_2 - W_1$
Average power	$N_{cp} = \frac{\Delta A}{\Delta t}$
Instant power	$N = \frac{dA}{dt} = Fv \cos \alpha$
Average angular speed	$\omega_{cp} = \frac{\Delta \varphi}{\Delta t}$
Instant angular speed	$\omega = \frac{d\varphi}{dt}$
Instant angular acceleration	$\varepsilon = \frac{d\omega}{dt}$
Rotation period	$T = \frac{2\pi}{\omega}$
Rotation frequency	$\nu = \frac{\omega}{2\pi}$
Equation of uniformly accelerated rotation	$\varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2}$
Angular speed of uniformly accelerated rotation	$\omega = \omega_0 + \varepsilon t$
Length of the arc is passed by a material point when moving in a circle	$S = \varphi \cdot R$
Linear and angular speed connection	$v = \omega \cdot R$
Tangential acceleration	$a_\tau = \varepsilon \cdot R$
Normal acceleration	$a_n = \omega^2 \cdot R$
Moment of inertia of the material point	$I = mR^2$
Moment of inertia of the solid body	$I = \sum_{i=1}^n m_i R_i^2$
Moment of impulse relative to the axis $z$	$L_z = I \cdot \omega$
Equation of the dynamics of rotational motion of the body relative to the fixed axis $z$	$M_z = \frac{dL_z}{dt} = I \cdot \varepsilon$
Steiner's theorem	$I = I_0 + ma^2$
Kinetic energy of rotational motion	$W_k = \frac{I\omega^2}{2}$
Work of torque forces	$A = M \cdot \varphi$
Instant power in rotational motion	$N = M \cdot \omega$

### Examples of solving problems

**Problem 1.** The motion of the body is given by the equation  $x = 2t^3 + t^2 + t - 1$ . Find the dependence of speed and acceleration on time.

**Given:**  $x = 2t^3 + t^2 + t - 1$

**Find:**  $a, v - ?$

Instant speed is found as a derivative of the time coordinate:

$$v = \frac{dx}{dt}, \quad v = 6t^2 + 2t + 1$$

Instant acceleration is the first derivative of speed by time:

$$a = \frac{dv}{dt}, \quad a = 12t + 2$$

**Problem 2.** The equation of motion of the material point along the axis is  $x = A + Bt + Ct^3$ , where  $A = 2\text{ m}$ ,  $B = 1\text{ m/s}$ ,  $C = -0,5\text{ m/s}^3$ . Find coordinate  $x$ , speed  $v_x$  and acceleration of the point  $a$  at the time  $t = 2\text{ s}$ .

**Given:**  $x = A + Bt + Ct^3$ ;  $A = 2\text{ m}$ ,  $B = 1\text{ m/s}$ ,  $C = -0,5\text{ m/s}^3$ .

**Find:**  $x, v_x, a - ?$

We find the coordinate by substituting the numerical values of coefficients  $A, B, C$  and time  $t$  in the equation of motion:

$$x = 2 + 1 \cdot 2 - 0,5 \cdot 8 = 0.$$

Instant speed is the first derivative of the time coordinate:

$$v_x = \frac{dx}{dt} = B + 3Ct^2.$$

The acceleration of the point is found by taking the first derivative of the velocity in time:

$$a = \frac{dv_x}{dt} = 6Ct.$$

At the time of  $t = 2\text{ s}$ :

$$v = 1 - 3 \cdot 0,5 \cdot 2^2 = -5\text{ m/s}.$$

$$a = 6 \cdot (-0,5) \cdot 2 = -6\text{ m/s}^2.$$

**Problem 3.** The cargo was dropped from a helicopter at an altitude of 300 m. After what time the cargo will reach the ground, if the helicopter: 1) stationary; 2) descends at a speed of 5 m / s; 3) rises at a speed of 5 m / s?

**Given:**  $h = 300\text{ m}$ ;  $v_x = 5\text{ m/s}$ .

**Find:**  $t_1, t_2, t_3 - ?$

1) If the helicopter is stationary, the distance in the vertical, which cargo passes in case of free fall:

$$h = \frac{gt^2}{2}.$$

Hence the time of the cargo drop to the ground:

$$t_1 = \sqrt{\frac{2h_0}{g}} = \sqrt{\frac{2 \cdot 300}{9,8}} = 7,8 \text{ s.}$$

2) If the helicopter drops at speed  $v_0$ , then the cargo drops along with the helicopter at speed  $v_0$ . Cargo movement equation:

$$h = v_0 t + \frac{gt^2}{2}.$$

When the cargo reaches the ground:

$$h = h_0, \quad t = t_2$$

$$t_2^2 + \frac{2v_0}{g} t_2 - \frac{2h_0}{g} = 0;$$

$$t_2 = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh_0}}{g}.$$

From here:

$$t_2 = -0,5 \pm 7,8 \text{ s.}$$

Discard  $t_2 < 0$  and get  $t_2 = 7,8 \text{ s.}$

3) If the helicopter rises at a speed, then the load has the same initial speed. At the time of reaching the earth  $h = h_0, \quad t = t_3$ .

Then:

$$h_0 = -v_0 t_3 + \frac{gt_3^2}{2}.$$

From here:

$$t_3 = \frac{v_0 \pm \sqrt{v_0^2 + 2gh_0}}{g} = \frac{5 \pm \sqrt{25 + 2 \cdot 9,8 \cdot 300}}{9,8} = (0,5 \pm 7,8) \text{ s.}$$

Discard  $t_3 < 0$  and get  $t_3 = 8,3 \text{ s.}$

**Problem 4.** The point travels in a circle with a radius of 20 cm with constant tangential acceleration  $a_\tau$ . Find the tangential acceleration of a point  $a_\tau$  if it is known that by the end of the fifth turn after the beginning of motion the linear speed of the point is 79,2 cm/s.

**Given:**  $R = 20$  cm;  $n = 5$ ;  $v = 79,2$  cm/s.

**Find:**  $a_\tau$  - ?

The linear speed  $v$  at uniformly accelerated motion in a circle is equal to ( $a_\tau = \text{const}$ ):

$$v = a_\tau t.$$

To find  $a_\tau$ , you need to know the time from the beginning of the rotation to the end of the 5th turn. It can be defined by using the ratio for angular displacement:

$$\Delta\varphi = \omega_0 t + \frac{\varepsilon t^2}{2}.$$

Given that the initial angular velocity is zero:

$$\Delta\varphi = \frac{\varepsilon t^2}{2} = 2\pi n.$$

Here  $\varepsilon$  is the angular acceleration,  $n$  is the number of rotations. Consequently:

$$t = \sqrt{\frac{4\pi n}{\varepsilon}}.$$

Angular acceleration is determined by the expression:

$$\varepsilon = \frac{a_\tau}{R}.$$

Then we get:

$$a_\tau = \frac{v}{t} = \frac{v}{\sqrt{\frac{4\pi n R}{a_\tau}}}.$$

So tangential acceleration:

$$a_\tau = \frac{v^2}{4\pi n R}.$$

Calculate its value:

$$a_\tau = 0,1 \text{ m/s}^2.$$

**Problem 5.** Ballistic pendulum is sometimes used to measure the velocity of balls, consisting of a massive freely suspended on a light rod by length of the body mass  $M$ , in which the bullet hits, getting stuck in it. The mass of the ball  $m$  deflects the pendulum from the equilibrium position at an angle  $\alpha$ . Find the speed of the ball if  $m = 20$  g,  $M = 5$  kg,  $l = 1$  m,  $\alpha = 60^\circ$ .

**Given:**  $m = 20$  g,  $M = 5$  kg,  $l = 1$  m,  $\alpha = 60^\circ$

**Find:**  $v - ?$

Apply the laws of conservation of momentum and energy to the pendulum - ball system. By the law of conservation of momentum for two bodies, given that the impact of the pendulum and the ball is inelastic, you can find the value of the joint speed of the pendulum and the ball after the pendulum hit the ball:

$$u = \frac{mv}{m + M}.$$

The law of conservation of energy relates the height  $h$  to which the pendulum rises, with the speed  $u$ :

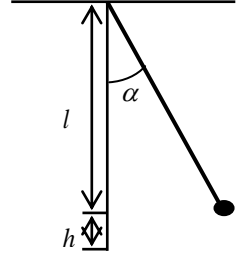
$$(M + m)gh = \frac{(M + m)u^2}{2}; \quad h = \frac{u^2}{2g}.$$

Given that the speed of the ball  $h = 2l \sin^2\left(\frac{\alpha}{2}\right)$  is determined by the ratio:

$$v = \frac{2(m + M)\sqrt{gl} \sin \frac{\alpha}{2}}{m} \approx \frac{2M\sqrt{gl} \sin \frac{\alpha}{2}}{m}.$$

Approximate equality is fair because  $m \ll M$ . After completing the calculation, we get:

$$v = 782,6 \text{ m.}$$



**Problem 6.** Between the two bodies masses  $m_1$  and  $m_2$  there is an inelastic blow, and the second body before impact was alone. Find a fraction of the kinetic energy that goes into heat.

**Given:**  $m_1, m_2, v_2 = 0$

**Find:**  $\frac{\Delta W}{W_1} - ?$

After the impact, both bodies move as a single unit with a common speed  $u$  equal to

$$u = \frac{m_1 v_1}{m_1 + m_2}.$$

Their kinetic energy will be:

$$W_2 = \frac{(m_1 + m_2)u^2}{2} = \frac{m_1^2 v_1^2}{2(m_1 + m_2)}.$$

Before the impact, the kinetic energy had only the first body:

$$W_1 = \frac{m_1 v_1^2}{2}.$$

The difference in these energies is equal to the amount of heat that will be released as a result of inelastic impact of bodies. Dividing this difference by the initial kinetic energy, we find the desired fraction of the kinetic energy converted to heat:

$$\eta = \frac{W_1 - W_2}{W_1} = 1 - \frac{W_2}{W_1} = \frac{m_1}{m_1 + m_2}.$$

**Problem 7.** The shot vertically upwards was made from a spring pistol. Determine the height  $h$  to which a bullet weighing 20 g will rise if the spring with a stiffness of 196 N/m was compressed before firing by 10 cm. The mass of the spring is neglected.

**Given:**  $m = 20$  g,  $k = 196$  N/m,  $x = 10$  cm.

**Find:**  $h - ?$

The Ball-Earth system (together with the gun) is a closed system in which conservative forces act – forces of elasticity and gravity. Therefore, the law of saving mechanical energy can be applied to solve the problem. According to this law, the full mechanical energy of the system  $E_1$  in the initial state (in this case before firing) is equal to the total energy  $E_2$  in the final state (when the ball rose to a height  $h$ ), that is:

$$E_1 = E_2, \text{ а } T_1 + \Pi_1 = T_2 + \Pi_2,$$

where  $T_1$  and  $T_2$  – kinetic energies of the system in the initial and final state;  $\Pi_1$  and  $\Pi_2$  are potential energies in the same states. Since the kinetic energy of the ball in the initial and final states is zero, this equality will be:

$$\Pi_1 = \Pi_2.$$

We take the potential energy of the ball in the field of gravity equal to zero at the level of placement of the gun. Then the potential energy of the system in the initial state is equal to the potential energy of the compressed spring:

$$\Pi_1 = \frac{kx^2}{2},$$

and in the final state - the potential energy of the ball at height  $h$ :

$$\Pi_2 = mgh.$$

Substituting the above expressions for energies in the formula for their equality for their equality we will receive:

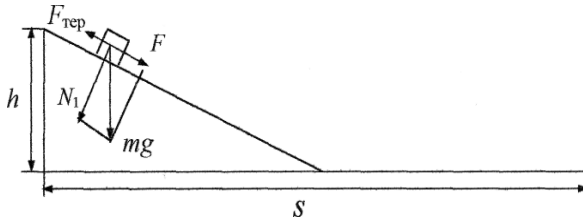
$$\frac{kx^2}{2} = mgh, \text{ and hence } h = \frac{kx^2}{2mg}.$$

After completing the calculation, we will receive  $h=5$  m.

**Problem 8.** The body slides off from the icy mountain height  $h$  and stops on an icy field at a distance  $s$  (in a horizontal direction) from the top of the mountain (see. figure). Determine the coefficient of friction  $k$ .

**Given:**  $h, s$ .

**Find:**  $k$  - ?



The body has only potential energy in the initial position  $E_1 = W_n = mgh$ . Full body energy in final position at stop time  $E_2 = 0$ . The change in body energy occurred due to the work of external forces. In this case, the external force is the force of friction. In the path segment along the inclined plane, its magnitude is equal to:

$$F_{mep} = kN_1 = kmg \cos \alpha.$$

Here the force of friction performs the work:

$$A_1 = -F_{mep}l = -F_{mep}h / \sin \alpha,$$

this work is negative because the force of friction is directed opposite to the direction of the body.

The friction force on the horizontal segment is equal to:

$$F'_{mep} = kmg,$$

and her work:

$$A_2 = -F'_{mep}(s-l) = -kmgs(s - h \cot \alpha).$$

Change of energy

$$E_2 - E_1 = -mgh$$

occurred due to the work of friction:

$$-mgh = -hkm\alpha g - kmg(s - h\alpha) .$$

From here:  $k = \frac{h}{s}$ .

**Problem 9.** Ignoring the friction, determine what work needs to be done to bring the flywheel, which weighs 0.2 tons, to a uniform rotation with a frequency of 100 rpm. approximately evenly distributed along its edge with a diameter of 1.2 m.

**Given:**  $M = 0,2 \text{ t}$ ,  $d = 1,2 \text{ m}$ ,  $\nu = 100 \text{ rpm}$ .

**Find:**  $A - ?$

The required work can be calculated as a change in the kinetic energy of the flywheel  $W_k$ . First, kinetic energy  $W_{k1} = 0$  and then reaches the value:

$$W_{k2} = \frac{I\omega^2}{2},$$

where  $I$  is the moment of inertia of the flywheel relative to the axis of rotation, and  $\omega$  is an angular speed of the flywheel.

Angular speed:

$$\omega = 2\pi\nu .$$

Therefore:

$$A = \Delta W = W_{k2} = 2I\pi^2\nu^2 .$$

The moment of inertia of the flywheel can be calculated by the formula:

$$I = mr^2 = \frac{md^2}{4} .$$

Substituting this expression in the formula for work, we find:

$$A = \frac{I}{2}m\pi^2\nu^2 ; A = 40 \text{ J} .$$

### Problems for self-solving

1.1. Dependence of the path traveled by the body from time  $t$  is given by the equation  $s = A - Bt + Ct^2$ , where  $A = 2 \text{ m}$ ,  $B = 3 \text{ m/s}$  and  $C = 4 \text{ m/s}^2$ . Find the value of speed and acceleration for  $t = 3 \text{ s}$ .

1.2. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = At - Bt^2 + Ct^3$ , where  $A = 2 \text{ m/s}$ ,  $B = 3 \text{ m/s}^2$  and  $C = 4 \text{ m/s}^3$ . Find the values of path and speed for  $t = 3 \text{ s}$ .

1.3. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = A - Bt + Ct^2$ , where  $A = 6 \text{ m}$ ,  $B = 3 \text{ m/s}$  and  $C = 2 \text{ m/s}^2$ . Find the value of speed and acceleration for  $t = 5 \text{ s}$ .

1.4. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = At - Bt^2 + Ct^3$ , where  $A = 6 \text{ m/s}$ ,  $B = 3 \text{ m/s}^2$  and  $C = 2 \text{ m/s}^3$ . Find the value of speed and acceleration for  $t = 5 \text{ s}$ .

1.5. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = A - Bt + Ct^2$ , where  $A = 6 \text{ m}$ ,  $B = 3 \text{ m/s}$  and  $C = 2 \text{ m/s}^2$ . Find the values of path and speed for  $t = 5 \text{ s}$ .

1.6. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = At - Bt^2 + Ct^3$ , where  $A = 2 \text{ m/s}$ ,  $B = 3 \text{ m/s}^2$  and  $C = 4 \text{ m/s}^3$ . Construct a graph of speed versus time for  $0 \leq t \leq 5$  after  $0.5 \text{ s}$ .

1.7. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = A - Bt + Ct^2$ , where  $A = 2 \text{ m}$ ,  $B = 3 \text{ m/s}$  and  $C = 4 \text{ m/s}^2$ . Construct a graph of path from time for  $0 \leq t \leq 5 \text{ s}$  after  $0.5 \text{ s}$ .

1.8. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = A - Bt + Ct^2$ , where  $A = 6 \text{ m}$ ,  $B = 3 \text{ m/s}$  and  $C = 2 \text{ m/s}^2$ . Construct a graph of acceleration from time for  $0 \leq t \leq 5 \text{ s}$  after  $1 \text{ s}$ .

1.9. Dependence of the path traveled by the body  $s$  from time  $t$  is given by the equation  $s = At - Bt^2 + Ct^3$ , where  $A = 6 \text{ m/s}$ ,  $B = 3 \text{ m/s}^2$  and  $C = 2 \text{ m/s}^3$ . Construct a graph of speed versus time for  $0 \leq t \leq 5 \text{ s}$  after  $1 \text{ s}$ .

1.10. A ball weighing  $10 \text{ kg}$  collides with a ball weighing  $4 \text{ kg}$ . The speed of the first ball is  $4 \text{ m/s}$ , the second  $12 \text{ m/s}$ . Considering the impact absolutely inelastic, find the speed of the balls after the impact in two cases: 1) when a small ball drives a large ball moving in the same direction; 2) when the balls move towards each other.

1.11. What is the highest speed a cyclist can develop by passing a rounding radius of  $50 \text{ m}$  if the coefficient of friction of slip between tires and asphalt is  $0.3$ ?

1.12. The car drives rounding the road with a curvature radius of  $200 \text{ m}$ . The coefficient of friction of the wheels in the road surface is  $0.1$  (icy road). At what speed will the car begin to demolish?

1.13. How far from the center of the Earth is the point at which the force of the full gravitational field of the Earth and the Moon is zero? Accept that the mass of the Earth is  $81$  times the mass of the Moon and that the distance from the center of the Earth to the center of the Moon is equal to  $60$  radii of the Earth.

1.14. At what linear speed will move the Earth's artificial satellite in circular orbit: 1) near the Earth's surface (air resistance to ignore); 2) at an altitude of  $200 \text{ km}$ ; 3) at an altitude of  $7000 \text{ km}$ ? Find the period of rotation of an artificial satellite around the Earth under these conditions.

1.15. Find the linear speed of movement of the Earth's orbit. Earth's orbit is considered circular.

1.16. Find the numerical value of the first cosmic speed, that is, the speed that must be given to the body near the surface of the Earth in a horizontal direction, so that it begins to move around the Earth in a circular orbit as its satellite.

1.17. The Moon's artificial satellite travels in a circular orbit at a distance of 20 km from the Moon's surface. Find the linear velocity of this satellite, as well as the period of its rotation around the Moon.

1.18. At what distance from the Earth's surface the acceleration of the force of gravity is equal to  $1 \text{ m/s}^2$ ?

1.19. The radius of the minor planet is 250 km; the average density is  $3 \text{ g/cm}^3$ . Determine the acceleration of free fall on the surface of the planet.

1.20. What will be the force of mutual attraction of two spacecraft masses of 10 t each if they approach a distance of 100 m?

1.21. Find the force of attraction between two protons that are at a distance of  $10^{-10} \text{ m}$  from each other. Proton mass is  $1,67 \cdot 10^{-27} \text{ kg}$ . Protons are considered point masses.

1.22. A disk with a radius of 40 cm rotates around the vertical axis. Cube stands on the edge of the disk. Find the number of revolutions per minute in which the cube slides off the disk. The coefficient of friction is 0.4.

1.23. The trolley stands on the floor in the form of a long board with light wheels. A person stands at one end of the board. The weight of the person is 60 kg, the weight of the board is 20 kg. At what speed (relative to the floor) will the trolley move if a person goes along the board at a speed (relative to the board)  $1 \text{ m/s}$ ? The mass of the wheels is neglected. Friction is not taken into account.

1.24. Find on condition of the previous task on which distance: 1) the trolley will move if the person moves to the other end of the board; 2) the person will move relative to the floor; 3) center of mass system trolley-person will move relative to the board, relative to the floor. The length of the board is 2 m.

1.25. How much weight should ballast be dropped from a uniformly lowered ballast so that it begins to rise evenly at the same speed? Weight of the balloon with ballast 1600 kg, lifting force of the balloon 11760 N. Air resistance force is considered the same when lifting and descending.

1.26. The body is thrown at an angle of  $30^\circ$  to the horizon weighing 5 kg with an initial speed of 20 m/s. Ignoring air resistance, find: 1) the impulse of force acting on the body during its flight; 2) change of body impulse during flight.

1.27. The axle with two disks located at a distance of 0.5 m from each other rotates at an angular speed corresponding to a frequency of 1600 rpm. The ball flying along the axis punches both discs; in this case, the hole from the ball in the second disk is shifted relative to the hole in the first disk at an angle  $\varphi = 12^\circ$ . Find the speed of the ball. Construct a graph of the angle dependence  $\varphi$  from the speed of movement of the ball for  $0,1 \nu \leq \nu \leq 2 \nu$ .

1.28. The point travels a circle radius of 10 cm with constant tangential acceleration. Find the tangential acceleration of the point if it is known that by the end of the fifth turn after the start of movement the speed of the point became

79,2 cm/s. Construct a graph of the dependence of the magnitude of speed of the material point from tangential acceleration for  $0,1 a_r \leq a_r \leq 2 a_r$ .

1.29. Construct a graph of the dependence of the height  $h$  on time  $t$  for the body, thrown vertically upwards with an initial speed of 9.8 m/s. Graph construct for the time interval from 0 to 2 s, that is, for  $0 \leq t \leq 2$  s, every 0.2 s. Air resistance is not taken into account.

1.30. The body mass of 0.5 kg moves so that the dependence of the path traveled by the body from the time of motion is given by the equation  $s = A \sin \omega t$ , where  $A = 5$  cm and  $\omega = \pi$  rad /s. Find the force acting on the body in  $1/6$  s after the start of the movement. Construct a graph of the dependence of the magnitude of the force acting on the body, from time to  $0 \leq t \leq 4$  s.

1.31. Trolley weighing 4 kg stands on the table. One end of the cord, thrown over the block, tied to the trolley. If mass of the kettlebell of 1 kg is tied to the other end of the cord, with what acceleration will the trolley move? Construct a graph of the dependence of the magnitude of the acceleration of the trolley from the mass of the kettlebell for  $0,5 \leq m \leq 6$  kg.

1.32. A wagon weighing 20 tons is moving with a constant negative acceleration, numerically equal to  $0,3 \text{ m/s}^2$ . The initial speed of the wagon is 54 km/h. 1) What braking force acts on the wagon? 2) How long will the wagon stop? 3) What distance will the wagon pass to the stop? Construct a graph of the dependence of the distance that the wagon will pass to the stop, from the mass of the wagon for  $0,1 m \leq m \leq 1,5 m$ .

1.33. Construct a graph of the dependence from the time of the kinetic and potential energy of the stone weighing 1 kg, thrown vertically up with an initial speed of 9.8 m/s, for  $0 \leq t \leq 2$  s through each 0,2 s. Air resistance is not taken into account.

1.34. Construct a graph of the dependence from the distance of the full energy of the stone under the condition of the previous problem.

1.35. Two balls are suspended on parallel threads of the same length so that they collide. The weight of the first ball is 0,2 kg, the weight of the second is 100 g. The first ball is rejected so that its center of weight rises to a height of  $h = 4,5$  cm and released. What is the height of the bullets after the collision, if: 1) impact elastic; 2) impact inelastic. Construct a graph of the dependence of the height of the rise of the second ball after the collision in two cases depending on the height of the lift of the center of mass of the first ball to a collision for  $1 \leq h \leq 10$  cm.

1.36. The kettlebell is suspended to a cord. The kettlebell was taken away aside so that the cord adopted a horizontal position, and released. What is the force of tension of the cord at the moment when the kettlebell passes the equilibrium position? What is the angle with the vertical cord at the moment when the tension force of the cord is equal to the weight of the kettlebell? Construct a graph of the

dependence on the tension force of the cord from the angle consisting of the vertical.

1.37. The length of a thin straight rod 60 cm, weight 100 g. Determine the moment of inertia of the rod relative to the axis, which is perpendicular to its length and passes through the point of the rod, 20 cm away from one of its ends. Construct a graph of the dependence of the magnitude of the moment of inertia of the rod from the distance for the interval:  $0 \leq l \leq 60$  cm.

1.38. The trolley went 5 m and reached a speed of 2 m/s under the influence of constant force. Determine the operation of the force if the weight of the trolley is 400 kg and the coefficient of friction is 0.01.

1.39. The boy stretched the spring to some length. In this position, the spring was intercepted by an adult man and stretched it for the same length.

1.40. Find the work you need to do to increase the speed of the body from 2 m/s to 6 m/s on the way 10 m. The constant force of friction acts all the way and is equal to 1,96 N. The body mass is 1 kg.

1.41. Calculate the work performed with uniformly accelerated lifting of a load weighing 100 kg to a height of 4 m in 2 s.

1.42. The work was carried out by a constant force of 78,4 J with vertical lifting of the load weighing 2 kg to a height of 1 m. With what acceleration was the load lifted?

1.43. The aircraft rises and at an altitude of 5 km reaches a speed of 360 km/h. How many times the work done when lifting against the weight force is greater than the work that goes on to increase the speed of the aircraft?

1.44. What work needs to be done to make a moving body of 2 kg: 1) increase its speed from 2 m/s to 5 m/s; 2) stop at the initial speed of 8 m/s?

1.45. The skater, standing on the ice, threw forward the kettlebell of 5 kg and, as a result of recoil, began to move back at a speed of 1 m/s. The weight of the skater is 60 kg. Determine the work done by the skater when throwing a kettlebell.

1.46. Find the power that develops the engine of the car weighing 1 t, if you know that the car is driving at a constant speed of 36 km/h: 1) horizontal road; 2) to the mountain with a slope of 5 m for every 100 m of the way; 3) down to the foot of the mountain with the same slope. The coefficient of friction is 0.07.

1.47. Determine the efficiency factor of the conveyor if it carries  $2 \cdot 10^7$  N of cargo per day from the ground level to the height of 5 m. Engine power 1,84 kW.

1.48. Find the work that needs to be done to compress the spring by 20 cm, if it is known that the force is proportional to the deformation and under the influence of force of 29,4 N the spring is compressed by 1 cm.

1.49. Find the numerical value of the second cosmic speed, that is, the velocity that must be given to the body near the surface of the Earth, so that it overcomes Earth's gravity and forever moves away from the Earth.

1.50. The neutron (the mass of which is  $m_0$ ) strikes the stationary nucleus of the carbon atom ( $m = 12m_0$ ). Find: how many times the kinetic energy of the neutron will decrease during the impact, considering the impact central and elastic.

1.51. Steel ball weighing 20 g, falling from a height of 1 m on a steel plate, bounces from it to a height of 81 cm. Find: 1) the pulse of force received by the plate during the impact; 2) the amount of heat released during the impact.

1.52. Metal ball weighing 20 g, falling from a height of 1 m on a steel plate, bounces from it to a height of 81 cm. Find the recovery factor of the ball material. The coefficient of recovery of body material is the ratio of the magnitude of body speed (velocity) after impact to its magnitude before impact.

1.53. The ball, flying horizontally, hits a body, suspended on a light solid rod, and gets stuck in it. Bullet weight 5 g, body weight 0.5 kg. Ball speed 500 m/s. At what limit length of the rod (distance from the point of suspension to the center of the body) the body from the impact of the ball will rise to the top point of its circular trajectory?

1.54. A disk with a mass of 1 kg and a diameter of 60 cm rotates around the axis through its center perpendicular to the plane, making 20 rps. What work needs to be done to stop the disk?

1.55. The ball with a diameter of 6 cm rolls without sliding along the horizontal plane, making 4 rps. Bullet weight 0.25 kg. Find the kinetic energy of the ball.

1.56. Disk weighing 2 kg rolls without sliding horizontal plane at a speed of 4 m/s. Find the kinetic energy of the disk.

1.57. The fan rotates at a speed corresponding to 900 rpm. The fan, rotating uniformly slowly, made 75 turns before stopping after switching off. The work of braking forces is 44.4 J. Find: 1) the moment of fan inertia; 2) the moment of the braking force.

1.58. From what slightest height should the cyclist come to the inertia (without friction) to pass the track, having the form of a “dead loop” radius of 3 m, and do not break away from the track at the top point of the loop. The mass of the cyclist together with the bicycle is 75 kg, moreover, the mass of the wheels is 3 kg. The wheels of the bicycle are considered hoops.

1.59. The hoop and the solid cylinder, having the same mass of 2 kg, roll without sliding at the same speed of 5 m/s. Find the kinetic energies of these bodies.

1.60. The ball rolls on the horizontal surface without slipping. Total kinetic energy of the ball 14 J. Determine the kinetic energy of translational and rotational motion of the ball.

1.61. The kinetic energy of a shaft rotating at a constant speed corresponding to 5 rps is equal to 60 J. Find the moment of movement of this shaft.

1.62. A copper ball with a radius of 10 cm rotates with speed corresponding to 2 rps, around the axis passing through its center. What work should be done to increase the angular speed of the ball rotation twice?

1.63. A constant tangential force of 19,62 N is applied to the rim of the disc mass 5 kg. What kinetic energy will the disk have in 5 s after the force starts?

1.64. The pencil, set vertically, falls on the table. What angular and linear speed will have the middle of the pencil at the end of the fall? The length of the pencil is 15 cm.

1.65. The flywheel, whose moment of inertia is  $40 \text{ kg/m}^2$ , began to rotate with uniform acceleration from a state of rest under the action of a moment of force of 20 N/m. Uniform accelerated rotation continued for 10 seconds. Determine the kinetic energy obtained by the flywheel.

1.66. The kinetic energy of the flywheel is 1000 J. Under the influence of a constant braking moment, the flywheel began to rotate equally and, having made 80 turns, stopped. Determine the moment of braking force.

1.67. Flywheel in the form of a disk with a mass of 80 kg and a radius of 30 cm is at rest. What work should be done to give the flywheel an angular speed of 10 rps? What work would have to be done if the disk had a smaller thickness but twice the radius at the same mass?

1.68. The horizontal platform with a mass of 100 kg rotates around the vertical axis passing through its center, making 10 rpm. A man weighing 588,6 N stands on the edge of the platform. If a person goes from the edge of the platform to its center, at what speed will the platform begin to rotate? Consider the platform a round uniform disk, and a person – a point mass.

1.69. A person stands on the edge of a horizontal platform that has the shape of a disk with a radius of 2 m. Platform weight is 200 kg, human weight is 80 kg. The platform can rotate around the vertical axis passing through its center. Ignoring friction, find at what angular speed the platform will rotate if a person goes along its edge at a speed of 2 m/s relative to the platform?

1.70. Ice with a cross-sectional area of  $1 \text{ m}^2$  and a thickness of 0.4 m floats in the water. What work needs to be done to immerse the ice in the water completely? The density of water is  $1000 \text{ kg/m}^3$ , of ice -  $900 \text{ kg/m}^3$ .

1.71. The boy rolls his hoop on a horizontal road at a speed of 7,2 km/h. At what distance  $s$  can the hoop roll on the hill due to its kinetic energy? The slope of the hill is 10 m for every 100 m of the way. Construct a graph of the dependence of the magnitude of the distance at which the hoop can roll on the hill, from the height  $h$  for the interval:  $1 \leq h \leq 50 \text{ m}$ .

1.72. Bullet mass 1 kg, rolling along the horizontal surface without friction and slip, hits the wall and rolls away from it. Ball speed before impact in the wall 10 cm/s, after the impact 8 cm/s. Find the amount of heat that was released when the impact. Construct a graph of the dependence of the magnitude of the amount of

heat released when impact, from the difference in speeds before and after impact in the interval:  $0 \leq \Delta v \leq 10 \text{ cm/s}$ .

1.73. On the drum radius of 20 cm, the moment of inertia is equal to  $0,1 \text{ kg/m}^2$ , the cord is wound, to which the load is tied with a mass of 0,5 kg. Before the start of rotation of the drum, the height of the load above the floor was 1m. When will the load fall to the floor? Construct a graph of the dependence of the magnitude of this time from the initial height of the load above the floor for  $0,5 \leq h \leq 5 \text{ m}$ . Friction should be neglected.

1.74. Ice sheet with a cross-sectional area of  $1 \text{ m}^2$  and a thickness of 0,4 m floats in the water. What work should be done to immerse the ice in the water completely? The density of water is  $1000 \text{ kg/m}^3$ , of ice is  $900 \text{ kg/m}^3$ . Construct a graph of the dependence of the magnitude of the work, which must be done, to immerse the ice in the water entirely, from the cross-section area of the ice sheet in the interval  $1 \leq s \leq 10 \text{ m}^2$ .

## SECTION 2. MOLECULAR PHYSICS AND THERMODYNAMICS

### Basic laws and formulas

Clapeyron's Law	$\frac{pV}{T} = \text{const} \ (m = \text{const})$
Boyle Mariotte's Law(Boyle's law)	$pV = \text{const} \ (T = \text{const}, m = \text{const})$
Gay-Lussac's Law	$\frac{V}{T} = \text{const} \ (p = \text{const}, m = \text{const})$
Charles' Law	$\frac{p}{T} = \text{const} \ (V = \text{const}, m = \text{const})$
Mendeleeev-Clapeyron equation	$pV = \frac{m}{M} RT$
The equation of state for a mixture of gases according to Dalton's law	$pV = \left( \frac{m_1}{M_1} + \frac{m_2}{M_2} + \dots + \frac{m_n}{M_n} \right) RT$
Main equation of kinetic gas theory for pressure	$p = \frac{1}{3} nm_0 \overline{v_{\text{сг}}^2} = \frac{2}{3} n \overline{E} = nkT$
First Law of Thermodynamics	$\Delta Q = \Delta U + \Delta A$
Gas operation against external pressure forces	$A = \int_{V_1}^{V_2} p dV, \ A = p(V_2 - V_1)$
Change of internal energy in isochoric process	$dU = \frac{m}{M} C_v dT$

Change in the amount of heat during the isobar process	$\delta Q = \frac{m}{M} C_p dT$
Mayer's equation	$R = C_p - C_v$
Work and the amount of heat in the isothermal process	$Q = A = \frac{m}{M} RT \ln \frac{V_2}{V_1}$
Poisson equation	$pV^\gamma = const, pT^{\frac{\gamma}{1-\gamma}} = const$ $VT^{\frac{1}{\gamma-1}} = const, TV^{\gamma-1} = const$
Work adiabatic process	$A = \frac{p_1 V_1^\gamma}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma})$ , $A = \frac{RT_1}{\gamma-1} \frac{m}{M} \left( 1 - \frac{T_2}{T_1} \right)$
Internal energy of an ideal gas	$U = \frac{i}{2} \frac{m}{M} RT$
Total energy of molecule motion	$\bar{E} = \frac{i}{2} kT$
The most likely speed	$v_i = \sqrt{\frac{2RT}{\mu}} = \sqrt{\frac{2kT}{m_0}}$
Average quadratic speed	$\bar{v}_{\kappa\theta} = \sqrt{\frac{3RT}{\mu}} = \sqrt{\frac{3kT}{m_0}}$
Average arithmetic speed	$\bar{v}_{ap} = \sqrt{\frac{8RT}{\pi\mu}} = \sqrt{\frac{8kT}{\pi m_0}}$
The law of distribution of molecules by speeds (Maxwell's law)	$dn = n_0 4\pi \left( \frac{m_0}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{m_0 v^2}{2kT}} v^2 dv$
Barometric formula	$p = p_0 e^{-\frac{\mu g}{RT} (h-h_0)} = p_0 e^{-\frac{m_0 g}{kT} (h-h_0)}$
Boltzmann distribution for concentration of molecules in potential field	$n = n_0 e^{-\frac{m_0 g}{kT} (h-h_0)} = n_0 e^{-\frac{\Delta W_n}{kT}}$

The average number of collisions of one molecule per unit time per unit volume	$\bar{z} = \sqrt{2}\pi d^2 n \bar{u}$
The average number of collisions of molecules per unit time per unit volume	$\bar{Z} = \frac{1}{\sqrt{2}} \pi d^2 n^2 \bar{u}$
Average free run length of gas molecules	$\bar{\lambda} = \frac{1}{\sqrt{2}\pi d^2 n}$
Fick's laws of diffusion	$dM = -D \frac{d\rho}{dx} dS dt$
Diffusion coefficient	$D = \frac{1}{3} \bar{u} \bar{\lambda}$
Fourier's law of heat conduction	$dQ = -K \frac{dT}{dx} dS dt$
Thermal conductivity coefficient	$K = \frac{1}{3} \bar{u} \bar{\lambda} c_v \rho$
Newton's law for the force of internal friction	$dF = -\eta \frac{dv}{dx} dS$
Coefficient of internal friction	$\eta = \frac{1}{3} \bar{u} \bar{\lambda} \rho$

### Examples of solving problems

**Problem 1.** In a vessel with a volume of 3 m<sup>3</sup> there is a mixture of 7 kg of nitrogen and 2 kg of hydrogen at a 27°C temperature. Determine the pressure and molar mass of the mixture of gases.

**Given:**  $V = 3 \text{ m}^3$ ;  $m_1 = 7 \text{ kg}$ ;  $M_1 = 28 \cdot 10^{-3} \text{ kg/mole}$ ;  $m_2 = 2 \text{ kg}$ ;

$M_2 = 2 \cdot 10^{-3} \text{ kg/mole}$ ;  $T = 300 \text{ K}$ .

**Find:**  $p$ ,  $M$  – ?

Write the Mendeleev-Clapeyron equation for nitrogen and hydrogen:

$$p_i V = \frac{m_i}{M_i} RT, \quad (1)$$

$$p_2 V = \frac{m_2}{M_2} RT, \quad (2)$$

where  $p_1$  is the partial pressure of nitrogen,  $m_1$  - the mass of nitrogen,  $M_1$  - its molar mass,  $V$  - the volume of the vessel,  $T$  - the temperature of the gas,  $R = 8,31 \frac{\text{Дж}}{\text{моль} \cdot \text{К}}$ ,  $p_2$  - the partial pressure of hydrogen,  $m_2$  - the mass of hydrogen,  $M_2$  - its molar mass.

According to Dalton's law, the pressure of the mixture is equal to the sum of partial pressures of the gases included in the mixture:

$$p = p_1 + p_2. \quad (3)$$

Under partial pressure  $p_1$  and  $p_2$ , we understand the pressure that gas would exert if it was only one in the vessel.

From equations (1) and (2) we find and substitute into equation

$$p = \frac{m_1 RT}{M_1 V} + \frac{m_2 RT}{M_2 V} = \left( \frac{m_1}{M_1} + \frac{m_2}{M_2} \right) \frac{RT}{V}. \quad (4)$$

The molar mass of the gas mixture is found by the formula:

$$M = \frac{m_1 + m_2}{\nu_1 + \nu_2}, \quad (5)$$

where  $\nu_1$  and  $\nu_2$  - the number of moles of nitrogen and hydrogen, respectively. The number of moles of gas we find by the formulas:

$$\nu_1 = \frac{m_1}{M_1}, \quad (6)$$

$$\nu_2 = \frac{m_2}{M_2}. \quad (7)$$

Substituting (6) and (7) in (5), we find:

$$M = \frac{m_1 + m_2}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}. \quad (8)$$

Substituting data in (3) and (4), we find the pressure of the mixture:

$$p = \left( \frac{7}{28 \cdot 10^{-3}} + \frac{2}{2 \cdot 10^{-3}} \right) \cdot \frac{8,31 \cdot 300}{3} = 1,04 \cdot 10^6 \text{ Pa}.$$

$$M = \frac{7 + 2}{\left( \frac{7}{28 \cdot 10^{-3}} + \frac{2}{2 \cdot 10^{-3}} \right)} = 7,2 \cdot 10^{-3} \text{ kg/mole}.$$

**Problem 2.** Determine the average kinetic energies of gradual and rotational motion of molecules contained in 4 kg of oxygen at a temperature of 200 K.

**Given:**  $m=4$  kg;  $M = 32 \cdot 10^{-3}$  kg/mole;  $T = 200$  K.

**Find:**  $\langle E_{nocm} \rangle, \langle E_{o\phi} \rangle - ?$

For a two-atom ideal gas, this is oxygen, the number of degrees of freedom of the molecule  $i = 5$ . On average, one degree of freedom is energy:

$$\langle \varepsilon_i \rangle = \frac{1}{2} kT,$$

where  $k$  - Boltzmann constant,  $T$  - thermodynamic temperature.

Of the five degrees of freedom, the gradual motion corresponds to three ( $i = 3$ ), and the rotational two ( $i = 2$ ). Then the energy of one molecule:

$$\langle \varepsilon_{nocm} \rangle = \frac{3}{2} kT, \quad \langle \varepsilon_{o\phi} \rangle = kT.$$

Number of oxygen molecules:

$$N = \nu \cdot N_A = \frac{m}{M} N_A,$$

where  $m$  - the mass of oxygen,  $M$  - its molar mass,  $\nu$  - the number of moles,  $N_A$  - Avogadro became.

Then the average kinetic energy of gradual movement of oxygen molecules:

$$\langle E_{nocm} \rangle = \frac{m}{M} N_A \cdot \frac{3}{2} kT = \frac{3}{2} \frac{m}{M} RT, \quad (1)$$

where  $R = kN_A$  - molar gas constant.

Average kinetic energy of rotational motion of oxygen molecules:

$$\langle E_{o\phi} \rangle = \frac{m}{M} RT. \quad (2)$$

Substituting numeric values in formula (1) and (2), we get:

$$\begin{aligned} \langle E_{nocm} \rangle &= \frac{3,4 \cdot 8,31 \cdot 200}{2,32 \cdot 10^{-3}} = 3,12 \cdot 10^5 \text{ J.} \\ \langle E_{o\phi} \rangle &= \frac{4 \cdot 8,32 \cdot 200}{32 \cdot 10^{-3}} = 2,08 \cdot 10^5 \text{ J.} \end{aligned}$$

**Problem 3.** Determine the average free run length of molecules and the number of co-strokes per 1 s occurring between all molecules of hydrogen, which is contained in a vessel of 1 liter at a temperature of 27°C and pressure of 104 Pa.

**Given:**  $V = 1\text{л} = 10^{-3}\text{м}^3$ ;  $M = 2 \cdot 10^{-3}\text{ kg/mole}$ ;  $T = 300\text{ K}$ ;  $P = 10^4\text{ Pa}$ ;  
 $d = 2,3 \cdot 10^{-10}\text{ м}$ ;

**Find:**  $\langle \lambda \rangle$ ,  $Z$  - ?

Average free path of molecules:

$$\langle \lambda \rangle = \frac{l}{\sqrt{2\pi} \cdot d^2 n}, \quad (1)$$

where  $d$  - effective diameter of the molecule,  $n$  - the concentration of molecules.

The concentration of  $n$  molecules is determined from the main equation of molecular - kinetic theory:

$$p = nkT,$$

where:

$$n = \frac{p}{kT}, \quad (2)$$

where  $k$  - Boltzmann's constant.

Substituting (2) in (1) we get:

$$\langle \lambda \rangle = \frac{kT}{\sqrt{2\pi} \cdot d^2 p}. \quad (3)$$

The number of co-strokes  $Z$  occurring between all molecules per 1 s is found from the ratio:

$$Z = \frac{\langle z \rangle N}{2}, \quad (4)$$

where  $N$  is the number of molecules of hydrogen in a vessel volume  $V = 10^{-3}\text{м}^3$ ;  
 $\langle z \rangle$  - the average number of co-strokes of one molecule per 1s.

Number of molecules in the vessel:

$$N = nV. \quad (5)$$

The average number of collisions of a molecule in 1s:

$$\langle z \rangle = \frac{\langle v \rangle}{\langle \lambda \rangle}, \quad (6)$$

where  $\langle v \rangle$  is the average arithmetic velocity of the molecule:

$$\langle v \rangle = \sqrt{\frac{8RT}{\pi \cdot M}}. \quad (7)$$

Substituting in (4) expressions (5), (6), (7), we find:

$$Z = \frac{1}{2} \frac{\sqrt{\frac{8RT}{\pi \cdot M}} \sqrt{2\pi} \cdot d^2 p \cdot}{kT} \cdot \frac{pV}{kT} = \frac{2\pi \cdot d^2 p^2 V}{(kT)^2} \sqrt{\frac{RT}{\pi \cdot M}}.$$

$$Z = \frac{2 \cdot 3,14 \cdot (2,3 \cdot 10^{-10})^2 \cdot (10^4)^2 \cdot 10^{-3}}{(1,38 \cdot 10^{-23} \cdot 300)^2} \cdot \frac{8,31 \cdot 300 \text{ K}}{3,14 \cdot 2 \cdot 10^{-3}} = 1,21 \cdot 10^{30} \text{ s}^{-1}.$$

$$\langle \lambda \rangle = \frac{1,38 \cdot 10^{-23} \cdot 300}{2 \cdot 3,14 \cdot (2,3 \cdot 10^{-10})^2 \cdot 10^4} = 1,76 \cdot 10^{-6} \text{ m}.$$

**Problem 4.** Determine the coefficient of diffusion and internal friction of helium, the temperature of which is 200 K and the pressure of 104 Pa.

**Given:**  $M = 4 \cdot 10^{-3} \text{ kg/mole}$ ;  $T = 200 \text{ K}$ ;  $p = 10^4 \text{ Pa}$ ;  $d = 1,9 \cdot 10^{-10} \text{ m}$

**Find:**  $D, \eta$  – ?

Diffusion coefficient:

$$D = \frac{l}{3} \langle v \rangle \langle \lambda \rangle, \quad (1)$$

where  $\langle \lambda \rangle$  - the average length of the free run of molecules;  $\langle v \rangle$  - the average arithmetic velocity of molecules, which is equal respectively:

$$\langle v \rangle = \sqrt{\frac{8RT}{\pi \cdot M}}, \quad (2)$$

$$\langle \lambda \rangle = \frac{kT}{\sqrt{2\pi} \cdot d^2 p}. \quad (3)$$

Substituting (2) and (3) in (1), we get:

$$D = \frac{l}{3} \sqrt{\frac{8RT}{\pi \cdot M}} \frac{kT}{\sqrt{2\pi} \cdot d^2 p} = \frac{2kT}{3\pi \cdot d^2 p} \sqrt{\frac{RT}{\pi \cdot M}}. \quad (4)$$

Coefficient of internal friction:

$$\eta = \frac{l}{3} \langle v \rangle \langle \lambda \rangle \rho, \quad (5)$$

where  $\rho$  is the density of gas at a temperature of 200 K and a pressure of  $10^4 \text{ Pa}$ .

To find  $\rho$ , we use the state equation of the ideal gas:

$$PV = \frac{m}{M} RT. \quad (6)$$

Considering, that  $\rho = \frac{m}{V}$ , we get:

$$\rho = \frac{PM}{RT} . \quad (7)$$

The coefficient of internal gas friction due to the diffusion coefficient is expressed by the ratio:

$$\eta = D\rho = \frac{DPM}{RT} . \quad (8)$$

Substituting the numerical values in (4) and (8), we obtain:

$$\begin{aligned} D &= \frac{2 \cdot 1,38 \cdot 10^{-23} \cdot 200}{3 \cdot 3,14 (1,9 \cdot 10^{-10})^2 \cdot 10^4} \sqrt{\frac{8,31 \cdot 200}{3,14 \cdot 4 \cdot 10^{-3}}} = \\ &= 5,9 \cdot 10^4 \text{ m}^2 / \text{s}. \\ \eta &= 5,9 \cdot 10^4 \frac{10^4 \cdot 4 \cdot 10^{-3}}{8,31 \cdot 200 \text{ K}} = 1,44 \cdot 10^5 \frac{\text{kg}}{\text{m} \cdot \text{s}}. \end{aligned}$$

**Problem 5.** The volume of argon under pressure 80 kPa increased from 1 liter to 2 liters. How will the internal energy of the gas change if the expansion was carried out: a) isobarically; b) adiabatically.

**Given:**  $V_1 = 1 \text{ l} = 10^{-3} \text{ m}^3$ ;  $V_2 = 2 \text{ l} = 2 \cdot 10^{-3} \text{ m}^3$ ;  $p = 0,8 \cdot 10^5 \text{ Pa}$ ;

$M = 40 \cdot 10^{-3} \text{ kg/mole}$ .

**Find:**  $\Delta U$  – ?

Let's use the first law of thermodynamics. According to this law, the amount of heat transferred to the system is spent on increasing internal energy  $\Delta U$  and performing external mechanical work  $A$  :

$$Q = \Delta U + A , \quad (1)$$

where:

$$\Delta U = \frac{m}{M} C_v \Delta T . \quad (2)$$

Here  $m$  - the mass of the gas,  $C_v$  - molar isochoric heat capacity, which is equal to:

$$C_v = \frac{i}{2} R . \quad (3)$$

Here  $i$  is the number of degrees of freedom. Then the expression (2) will look like:

$$\Delta U = \frac{m}{M} \frac{i}{2} R \Delta T . \quad (4)$$

Let's write the Clapeyron-Mendeleev equation for the initial and final states of the gas in the isobar process:

$$pV_1 = \frac{m}{M} RT_1 \text{ and } pV_2 = \frac{m}{M} RT_2 ,$$

where

$$p(V_2 - V_1) = \frac{m}{M} R(T_2 - T_1). \quad (5)$$

Substituting (5) in (4), we get:

$$\Delta U = \frac{i}{2} P(V_2 - V_1). \quad (6)$$

Heat exchange with the external environment is absent with adiabatic gas expansion, therefore  $Q = 0$ . Equation (1) writes as:

$$\Delta U - A = 0, \quad (7)$$

or

$$A = -\Delta U. \quad (8)$$

The “minus” sign before  $\Delta U$  means that the gas expansion work can only be done by reducing the internal energy of the gas.

Work carried out by gas in the adiabatic process:

$$A = \frac{P_1 V_1}{\gamma - 1} \left[ 1 - \left( \frac{V_1}{V_2} \right)^{\gamma - 1} \right], \quad (9)$$

where  $\gamma = \frac{C_0}{C_V} = \frac{i + 2}{i}$  - adiabat indicator. For single-atom gas argon 3. Therefore

$$\gamma = \frac{3 + 2}{3} = 1,67.$$

We find the change of internal energy in the adiabatic process for argon, given the formula (8) and (9):

$$\Delta U = \frac{P_1 V_1}{(\gamma - 1)} \left[ \left( \frac{V_1}{V_2} \right)^{\gamma - 1} - 1 \right] \quad (10)$$

Substituting numeric values in (6) and (10), we get:

a) with isothermal expansion:

$$\Delta U = \frac{3}{2} \cdot 0,8 \cdot 10^5 \cdot (2 \cdot 10^{-3} - 10^{-3}) = 120 \text{ J.}$$

b) with adiabatic extension:

$$\Delta U = \frac{0,8 \cdot 10^5 \cdot 10^{-3}}{(1,67 - 1)} \left[ \left( \frac{10^{-3}}{2 \cdot 10^{-3}} \right)^{1,67 - 1} - 1 \right] = -44,6 \text{ J.}$$

### Problems for self-solving

2.1. 14 g of nitrogen under pressure  $10^5 \text{ N/m}^2$  and at a temperature of  $27^\circ\text{C}$  is in a closed vessel. The pressure in the vessel increased 5 times after heating. Find: 1) to what temperature was heated gas; 2) what is the volume of the vessel?

2.2. 14 g of nitrogen is under a pressure  $10^5 \text{ N/m}^2$  and at a temperature of  $27^\circ\text{C}$  is in a closed vessel. The pressure in the vessel increased 5 times after heating. Find the amount of heat given to the gas?

2.3. How much heat must be given to 12g of oxygen to heat it to  $50^\circ$  at constant pressure?

2.4. 10 g of oxygen is under a pressure of  $3 \cdot 10^5 \text{ N/m}^2$  at a temperature of  $10^\circ\text{C}$ . After heating at constant pressure, the gas took up a volume of 10 liters. Find: 1) the amount of heat obtained by gas; 2) change of internal energy of gas.

2.5. 10 g of oxygen is under a pressure of  $3 \cdot 10^5 \text{ N/m}^2$  at a temperature of  $10^\circ\text{C}$ . After heating at constant pressure, the gas took up a volume of 10 liters. Find the work done by the gas during expansion.

2.6. What is the air pressure at the top of the TV tower at an altitude of 533 m, if the pressure at its foot is 101,3 kPa, temperature  $280 \text{ K}$ ?

2.7. The barometer shows 78,6 kPa in the cabin of the aircraft. At what altitude does the plane fly, if the airfield barometer showed 0.1 MPa? Air temperature is  $278 \text{ K}$ .

2.8. At what altitude does the aircraft fly if the barometer in the cockpit showed the pressure of 100,6 kPa at the airfield, and during the lift of the aircraft, its performance decreased by 11,3 kPa? Air temperature is  $290 \text{ K}$ .

2.9. At what altitude will the concentration of air molecules halve above sea level? Air temperature is  $17^\circ\text{C}$ .

2.10. The height of the mountain is 2 km. Determine the pressure and concentration of air molecules at the top of the mountain, if the pressure at its foot is 101 kPa and the temperature is  $10^\circ\text{C}$ .

2.11. Determine the average free path length of carbon dioxide molecules at a temperature of  $100^\circ\text{C}$  and a pressure of 0,1 mm of mercury. The diameter of the carbon dioxide molecule is equal to  $32 \cdot 10^{-8} \text{ cm}$ .

2.12. Using an ionization manometer mounted on an artificial satellite of the Earth, it was found that at an altitude of 300 km from the Earth's surface in  $1 \text{ cm}^3$  of the atmosphere is about a billion particles of gas. Find the average free path length of gas particles at this height. The diameter of the particles is taken to be equal to  $2 \cdot 10^{-10} \text{ m}$ .

2.13. Find the average free path of air molecules under normal conditions. The diameter of the air molecule is assumed to be equal to  $3 \cdot 10^{-10} \text{ m}$ .

2.14. Find the average number of collisions per 1s of carbon dioxide molecules at a temperature of  $100^\circ\text{C}$ , if the average free path length under these conditions is  $8,7 \cdot 10^{-2} \text{ cm}$ .

2.15. Find the average number of collisions per 1s of nitrogen molecules at a temperature of  $27^\circ\text{C}$  and a pressure of 400 mm of mercury.

2.16. Oxygen is in a 0,5 liter vessel under normal conditions. Find the total number of collisions between oxygen molecules in this volume in 1s.

2.17. Find the average free path length of helium atoms under conditions when the density of helium is  $2,1 \cdot 10^{-2} \text{ kg/m}^3$ .

2.18. What is the average free run length of hydrogen molecules at a pressure of  $10^{-3} \text{ mm.}$  of mercury and temperature  $50^\circ\text{C}$ ?

2.19. What is the total kinetic energy (gradual and rotational motion) of molecules of one mole of oxygen ( $\text{O}_2$ ) at 300 K?

2.20. Determine the kinetic energy of the gradual motion, the complete kinetic energy of the water vapor molecule at a temperature of 573 K and the sum of the kinetic energies of all molecules of one mole of vapor.

2.21. Determine the energy of gradual and rotational motion of 1 kg of oxygen molecules at a temperature of 280 K.

2.22. Find the diffusion coefficient of hydrogen under normal conditions, if the average free path of molecules under these conditions is  $1,6 \cdot 10^{-7} \text{ m}$ .

2.23. Find the diameter of the oxygen molecule, if it is known that the oxygen coefficient of internal friction at  $0^\circ\text{C}$  is  $18,8 \cdot 10^{-6} \text{ N}\cdot\text{s/m}^2$ .

2.24. Find the coefficient of thermal conductivity of hydrogen, if it is known that the coefficient of internal friction for it under these conditions is equal to  $8,6 \cdot 10^{-6} \text{ N}\cdot\text{s/m}^2$ .

2.25. Find the coefficient of thermal conductivity of air at a temperature of  $10^\circ\text{C}$  and a pressure of  $105 \text{ N/cm}^2$ . The diameter of the air molecule should be equal to  $3 \cdot 10^{-8} \text{ cm}$ .

2.26. Calculate the change in entropy  $5 \cdot 10^{-3} \text{ kg}$  of hydrogen, which isothermally expands from  $10^{-2} \text{ m}^3$  to  $2,5 \cdot 10^{-2} \text{ m}^3$ .

2.27. How many times is the average free path of air molecules under normal conditions greater than the average distance between molecules? The diameter of air molecules should be equal to  $0,3 \text{ nm}$ .

2.28. Oxygen gas is under normal conditions. Calculate the average number of collisions experienced by each oxygen molecule per unit time and the number of total collisions of molecules in  $1 \text{ m}^3$  per 1s.

2.29. Determine the limit concentration of helium molecules in a spherical 10 cm diameter flask, in which collisions between molecules in the path equal to the diameter of the flasks will not occur.

2.30. How many collisions does a neon molecule experience in 1s at a temperature of 600 K under a pressure of  $1,3 \cdot 10^2 \text{ Pa}$ ?

2.31. Estimate the hydrogen pressure in a vessel with a capacity of 1 liter, at which the free path of molecules becomes larger than the characteristic size of the vessel. The temperature of hydrogen is 300 K.

2.32. The tank contains water vapor at a temperature of 400 K and a pressure of 1,3 Pa. How many collisions occur every second between vapor molecules of  $1 \text{ m}^3$ ?

2.33. The average free run length of helium molecules at some pressure and a temperature of  $22^{\circ}\text{C}$  is  $0,1\ \mu\text{m}$ . After isothermal compression, the gas pressure increased by 2 times. Determine the average number of collisions of helium molecules per unit volume per unit time after the end of the process.

2.34. Find the change in entropy when converting 10 g of ice at  $20^{\circ}\text{C}$  to steam at  $100^{\circ}\text{C}$ .

2.35. Find the entropy increase when converting 1 g of water at  $0^{\circ}\text{C}$  to steam at  $100^{\circ}\text{C}$ .

2.36. Find the change in entropy when melting 1 kg of ice at  $0^{\circ}\text{C}$ .

2.37. 640 g of molten lead at the melting point was poured on ice at  $0^{\circ}\text{C}$ . Find the change in entropy in this process.

2.38. Find the change in entropy at the transition of 8 g of oxygen from a volume of 10 liters at a temperature of  $80^{\circ}\text{C}$  to a volume of 40 liters at a temperature of  $300^{\circ}\text{C}$ .

2.39. What is the temperature of 2 g of nitrogen, which occupies a volume of  $820\ \text{cm}^3$  at a pressure of 2 atm? Consider gas as: 1) ideal; 2) real.

2.40. What is the temperature of 3,5 g of oxygen, which occupies a volume of  $90\ \text{cm}^3$  at a pressure of 28 atm? Consider gas as 1) ideal; 2) real.

2.41. 10 g of helium occupy a volume of  $100\ \text{cm}^3$  at a pressure of  $108\ \text{N/m}^2$ . Find the temperature of the gas, considering it as: 1) ideal; 2) real.

2.42. 1 kmol of carbon dioxide is at a temperature of  $100^{\circ}\text{C}$ . Find the gas pressure, considering it: 1) real; 2) ideal. Solve the problem for a volume of  $1\ \text{m}^3$ .

2.43. Find the specific heat at constant pressure of the following gases: 1) hydrogen chloride; 2) neon; 3) nitric oxide; 4) carbon monoxide; 5) mercury vapor.

2.44. 6 kg of oxygen at a constant temperature of 400 K is compressed from 0,3 MPa to 3 MPa. Determine the initial, final volumes occupied by the gas, and the work done during its compression. Construct a dependency graph of the work done during gas compression from the pressure difference in the initial and final state in the range from 0 to 2,7 MPa.

2.45. Every hour the compressor compresses  $150\ \text{m}^3$  of air increasing its pressure from 0,1 to 0,7 MPa. How much heat must be removed from the compressor cylinder to compress the air isothermally? Construct a dependency graph of this amount of heat on the pressure difference in the initial and final state in the interval from 0 to 0,6 MPa.

2.46. How will the internal energy of 0,1 kg of oxygen change if it is heated from 283 K to 333 K at constant pressure? Construct a dependency graph of the change in this internal energy on the temperature difference in the initial and final state in the range from 0 to 50 K.

2.47. As a result of adiabatic compression of 1 mole of diatomic gas, its temperature increased by  $7^{\circ}\text{C}$ . What is the work done? Construct a dependency

graph on the magnitude of this work on the temperature difference in the initial and final state in the interval  $0 \leq \Delta t \leq 50^\circ\text{C}$ .

2.48. How will the internal energy of 0,1 kg of oxygen change if it is heated from 283 K to 333 K at a constant volume? Construct a change dependency graph of this internal energy from the temperature difference in the initial and final state in the interval  $0 \leq \Delta t \leq 50^\circ\text{C}$ .

2.49. For some diatomic gas, the specific heat at constant pressure is  $3,5 \text{ cal}/(\text{g}\cdot\text{deg})$  ( $1 \text{ cal} = 4,19 \text{ J}$ ). Why is the mass of one kilomole of this gas? Construct diagrams of specific heat at constant pressure for gases with different number of atoms in the molecule.

2.50. 10 g of oxygen are under a pressure of  $3 \cdot 10^5 \text{ N/m}^2$  at a temperature of  $10^\circ\text{C}$ . After heating at constant pressure, the gas took up a volume of 10 liters. Find: 1) the amount of heat generated by gas; 2) the energy of thermal motion of gas molecules before and after heating. Construct a dependency graph of the amount of heat received by the gas, from the volume occupied by it after heating, in the interval  $10 \leq V \leq 100$  liters.

2.51. Under some conditions, the average free path length of carbon dioxide molecules is  $1,6 \cdot 10^{-7} \text{ m}$  and the average arithmetic velocity of its molecules is 1,95 km/s. Why is the average number of collisions per 1 s of molecules of this gas, if at the same temperature the gas pressure is reduced by 1,27 times? Construct a dependency graph of the average number of collisions per 1 s of carbon dioxide molecules, from the pressure in the interval  $10^4 \leq p \leq 10^5 \text{ Pa}$ , at a constant temperature of 300 K.

2.52. Calculate the average kinetic energy of translational motion and the total average kinetic energy of molecules at a temperature of 273 K for monoatomic, diatomic and polyatomic gases. The answers compare. Construct dependency graphs of the average kinetic energy of gradual motion and the total average kinetic energy of molecules for single, diatomic and multiatomic gases from temperature in the interval  $200 \leq T \leq 1000 \text{ K}$  in one coordinate system.

2.53. Carbon dioxide and nitrogen are at the same temperature and pressure. Find the ratios for these gases: 1) diffusion coefficients; 2) coefficients of internal friction; 3) thermal conductivity coefficients. The diameters of the molecules of these gases are considered to be the same. Construct dependency graphs of transfer coefficients from temperature for data in the interval  $200 \leq T \leq 1000 \text{ K}$  in one coordinate system. The pressure of gases should be considered normal and constant.

2.54. The nitrogen molecule under normal conditions moves at a speed of 454 m/s. Determine the momentum of the molecule. Construct a dependency graph of the magnitude of the momentum of the molecule from the temperature in the interval  $200 \leq T \leq 1000 \text{ K}$ .

2.55. Find the most likely, average quadratic, average arithmetic velocity of chlorine molecules at a temperature of 500 K. Construct dependency graphs of the magnitude of these velocities of the molecule from the temperature in the interval  $200 \leq T \leq 1000$  K.

2.56. Find the average arithmetic velocity of gas molecules when it is known that the average quadratic velocity of their 400 m/s. Construct a dependency graph of the magnitude of the average arithmetic velocity of carbon dioxide molecules from the temperature in the interval  $200 \leq T \leq 1000$  K.

2.57. Determine the coefficient of performance of the engine of a passenger car if its engine develops an effective power of 47 kW and consumes 299 g of gasoline per 1 kW hourly. Construct a dependency graph of the coefficient of performance of the engine from mass of gasoline consumed per hour per 1 kW in the interval  $150 \leq m \leq 400$  g.

2.58. Determine the average effective power developed by the engine of a car that consumes 0,36 liters of gasoline per 1 km of road, if its efficiency is 28% and speed is 80 km/h. Construct a dependency graph of the average effective power of the engine from the volume of gasoline consumed per 1 km in the interval  $0,3 \leq V \leq 1,5$  l.

2.59. A modern truck has a 155 kW engine. Determine the fuel consumption per 1 km of road, if at a speed of 80 km/h it develops at full power. Engine efficiency is 30%. Fuel combustion heat  $q = 4,27 \cdot 10^7$  J/kg. Construct a dependency graph of fuel consumption on 1 km of the road from the speed of movement in the interval  $50 \leq v \leq 150$  km/h.

2.60. Due to the isobaric expansion of one mole of helium, its volume increases 4 times. Find the entropy change. Construct a dependency graph of the entropy change from the amount of increase in volume in the interval  $2 \leq V_2 / V_1 \leq 10$  times.

2.61. Gas at a temperature of 298 K and a pressure of  $0,5 \cdot 10^5$  Pa occupies a volume of  $2 \cdot 10^{-2}$  m<sup>3</sup>. Determine the change in entropy if the gas is isothermally compressed to a volume of  $5,7 \cdot 10^{-3}$  m<sup>3</sup>. Construct a dependency graph of the entropy change from the amount of volume change in the interval from  $2 \cdot 10^{-2}$  m<sup>3</sup> to  $6 \cdot 10^{-3}$  m<sup>3</sup>.

2.62. 0,16 kg of oxygen is heated from 323 K to 333 K. Find the amount of heat given to oxygen and the change in internal energy if the process takes place at constant volume. Construct dependency graphs of the amount of heat given to oxygen and the change in internal energy from the change in temperature  $T_2 - T_1$  with a change  $T_2$  in the interval from 300 K to 400 K in one coordinate system.

2.63. 0,16 kg of oxygen is heated from 323 K to 333 K. Find the amount of heat given to oxygen and the change in internal energy if the process takes place at

constant pressure. Construct dependency graphs of the amount of heat given to oxygen and the change in internal energy from the change in temperature  $T_2 - T_1$  with a change  $T_2$  in the interval from 300 K to 400 K in one coordinate system.

2.64. A heat engine that operates on the Carnot cycle and performs 28,05 kJ in one cycle. Determine the efficiency of the cycle, the amount of heat  $Q_1$  and  $Q_2$ , respectively, received from the heater and transferred to the refrigerator in one cycle, if the temperature of the heater is 150°C, the refrigerator temperature is 10°C.

2.65. The efficiency of an ideal heat engine is 25%. What will be the cooling coefficient  $\varepsilon$  of this machine if you make it work on the reverse cycle of Carnot as a refrigeration machine?

2.66. Due to adiabatic compression, the air pressure was increased from 50 kPa to 0,5 MPa. Then at constant volume the air temperature was reduced to the initial. Determine the gas pressure at the end of the process.

2.67. What part of the heat received from the heater is given to the refrigerator in the direct Carnot cycle, if the temperature of the heater is 500 K and the temperature of the refrigerator is 175 K?

2.68. Find the efficiency of a cycle consisting of two isobars and two adiabats, if the temperatures of the characteristic points are equal:  $T_1 = 370$  K,  $T_2 = 600$  K,  $T_3 = 500$  K,  $T_4 = 350$  K. Explain the solution with the diagram  $p - V$ .

2.69. Due to the 1 kJ of heat received from the heater, the machine operating on the Carnot cycle performs 0,5 kJ. Heater temperature 500 K. Determine the temperature of the refrigerator.

2.70. The heat engine performs 200 J in a direct Carnot cycle. Heater temperature 375 K, refrigerator 300 K. Find the amount of heat that keeps the machine from the heater.

2.71. Determine how many percent the efficiency of the Carnot direct cycle will change if the refrigerator temperature decreases from 494 K to 394 K.

2.72. Carrying out a direct Carnot cycle, the gas gives the refrigerator 25% of the heat received from the heater. Determine the temperature of the refrigerator if the heater temperature is 500 K.

2.73. The heat engine operates on the Carnot cycle, the efficiency of which is 25%. What will be the efficiency of this machine if it performs the same cycle in the opposite direction?

2.74. Determine the operation of an ideal heat engine in one cycle if it receives 2095 J of heat from the heater during the cycle. Temperature of the heater is 500 K, the refrigerator is 300 K.

2.75. The temperature of the heater of the heat engine operating on the Carnot cycle is 480 K, the temperature of the refrigerator is 390 K. What should be the temperature of the heater at the same temperature of the refrigerator, so that the efficiency of the machine doubles?

## SECTION 3. ELECTROSTATICS

### Basic laws and formulas

Coulomb's law	$F = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$
Electrostatic field strength	$E = \frac{F}{q}$
Electrostatic field strength vector flow	$\Phi_E = E \cdot S \cdot \cos \alpha$
Ostrogradsky-Gauss theorem for electrostatic field strength	$\Phi_E = \frac{1}{\epsilon\epsilon_0} \sum_{i=1}^n q_i$
Electrostatic field strength of a point charge	$E = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{r^2}$
Electrostatic field strength of a uniformly charged sphere of radius $R$ at a distance $r$ from the center of the sphere: a) inside the sphere ( $r < R$ ); b) on the surface of the sphere ( $r = R$ ); c) outside the sphere ( $r > R$ )	<p>a) <math>E = 0</math></p> <p>b) <math>E = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{R^2}</math></p> <p>c) <math>E = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{r^2}</math></p>
Principle of superposition (overlay) of electrostatic fields	$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n$
Electrostatic field strength created by an infinitely long uniformly charged filament	$E = \frac{1}{2\pi\epsilon\epsilon_0} \cdot \frac{\tau}{r}$
Linear charge density	$\tau = \frac{dq}{dl}$
Field strength created by an infinite evenly charged plane	$E = \frac{\sigma}{2\epsilon\epsilon_0}$
Surface charge density	$\sigma = \frac{dq}{dS}$
Electrical displacement	$D = \epsilon\epsilon_0 E$
Electric field potential	$\varphi = \frac{W_{II}}{q}$

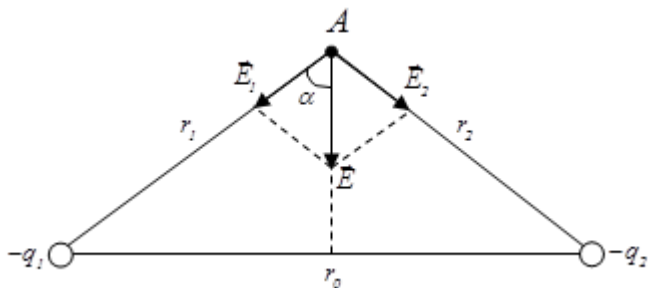
Potential of an electrostatic field of point charge	$\varphi = \frac{q}{4\pi\epsilon\epsilon_0 r}$
Potential of an electrostatic field of a uniformly charged sphere of radius $R$ , at a distance $r$ from the center of the sphere: a) inside the sphere ( $r < R$ ); b) on the surface of the sphere ( $r = R$ ); c) outside the sphere ( $r > R$ )	a) $\varphi = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{R}$ b) $\varphi = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{R}$ c) $\varphi = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{r}$
Potential of an electrostatic field created by a system of point charges	$\varphi = \varphi_1 + \varphi_2 + \dots + \varphi_n$
Relationship of electrostatic field strength with potential	$\vec{E} = \vec{\text{grad}} \varphi,$ $E = \frac{\varphi_1 - \varphi_2}{d} = \frac{U}{d}$
Work of the electrostatic field to move the point charge	$A = q(\varphi_1 - \varphi_2)$
Electrical capacity of the capacitor	$C = \frac{q}{U}$
Electrical capacity of a flat capacitor	$C = \frac{\epsilon\epsilon_0 S}{d}$
Energy of a charged capacitor	$W = \frac{CU^2}{2} = \frac{q^2}{2C} = \frac{qU}{2}$
Volumetric energy density of an electrostatic field	$\omega = \frac{\epsilon\epsilon_0 E^2}{2} = \frac{ED}{2}$

### Examples of solving problems

**Problem 1.** Two identical negative charges of 9 nC each are in water at a distance of 8 cm from each other. Determine the field strength and potential at a point located at a distance of 5 cm from each of the charges.

**Given:**  $q_1 = q_2 = 9 \text{ nC}$ ;  $r_0 = 8 \text{ cm}$ ;  $r_1 = r_2 = 5 \text{ cm}$

**Find:**  $E, \varphi$ —?



The field strength at point A, which is created by the charges  $q_1$  and  $q_2$  according to the principle of superposition of the fields, is equal to the vector sum of the electric field strengths created by each of the charges separately:

$$\vec{E} = \vec{E}_1 + \vec{E}_2. \quad (1)$$

By the cosine theorem,

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos 2\alpha}. \quad (2)$$

Point charge field strength  $q$ :

$$E = \frac{1}{4\pi\epsilon\epsilon_0} \cdot \frac{q}{r^2},$$

where  $\epsilon$  is the dielectric constant;  $\epsilon_0$  - electric constant;  $r$  is the distance from the charge to the point of the field at which its intensity is determined. The charges  $q_1$  and  $q_2$  are negative, so the vectors are directed along the lines of tension to the charges. Under the condition of the problem, the charges  $q_1 = q_2$  and are located at the same distance from point A, therefore. Thus, formula (2) will look like:

$$E = 2E_1 \cos \alpha,$$

where  $\cos \alpha = h / r_1$ ;  $h = \sqrt{r_1^2 - r_0^2} / 2$ .

$$h = \sqrt{(5 \cdot 10^{-2})^2 - (4 \cdot 10^{-2})^2} = 3 \cdot 10^{-2} \text{ m}.$$

Therefore, the tension at point A will be:

$$E = \frac{2Q_1 \cdot h}{4\pi\epsilon\epsilon_0 r_1^3}. \quad (3)$$

Calculate:

$$E = \frac{2 \cdot 9 \cdot 10^{-9} \cdot 3 \cdot 10^{-2}}{4 \cdot 3,14 \cdot 81 \cdot 8,85 \cdot 10^{-12} \cdot (0,05)^3} = 480 \text{ V / m}.$$

The potential  $\varphi$  created by a system of point charges at a given point in the field is equal to the algebraic sum of the potentials generated by each of the

charges:  $\varphi = \sum_{i=1}^n \varphi_i$ . Therefore, the potential  $\varphi$  of the resulting field at point A is equal to  $\varphi = \varphi_1 + \varphi_2$ . Since the potential of the field created by the point charge is equal to:  $\varphi = q / 4\pi\epsilon_0 r$ , then we obtain:

$$\varphi = \varphi_1 + \varphi_2 = \frac{q_1}{4\pi\epsilon_0 r_1} + \frac{q_2}{4\pi\epsilon_0 r_2} = \frac{2q_1}{4\pi\epsilon_0 r_1}. \quad (4)$$

Calculate:

$$\varphi = \frac{2 \cdot 9 \cdot 10^{-9}}{4 \cdot 3,14 \cdot 81 \cdot 8,85 \cdot 10^{-12} \cdot 5 \cdot 10^{-2}} = -40 \text{ V}.$$

Substituting the numerical values of  $r$  in formulas (3) and (4), we can construct graphical dependences of these quantities.

**Problem 2.** A positive charge  $q = 5,0 \cdot 10^{-9}$  C is evenly distributed along a thin wire ring of radius  $R = 10$  cm. Find the electric field strength on the axis of the ring at a point located at a distance  $L$  from the center of the ring.

Let's take the element of the ring  $dl$ . The charge  $dq$  that is on it can be considered a point. Then the electric field strength at the point created by this element  $dl$ :

$$dE = \frac{dq}{4\pi \cdot \epsilon \cdot \epsilon_0 r^2}.$$

It is directed along the radius vector  $\vec{r}$ , from the ring element to the point where we find the tension.

According to the principle of superposition, the resulting voltage at a point is equal to the vector sum of the field strengths created at that point by each element of the charge ring  $dq$ . Express the vector  $d\vec{E}$  through the components  $d\vec{E}_r$  directed along the axis of the ring, and  $d\vec{E}_n$  directed perpendicular to the axis.

$$d\vec{E} = d\vec{E}_r + d\vec{E}_n.$$

The components  $d\vec{E}_n$  of each of the two diametrically opposed elements are mutually compensated. This is because for each charge element  $dq$  there is a diametrically opposite charge element  $dq'$ , therefore

$$d\vec{E}_n + d\vec{E}_n' = 0.$$

Therefore, the resulting stress at the point is equal to the sum of the components of the stresses along the axis of the ring, and the modulus of tension is equal to the sum of the modules of these components, since their directions are the same.

$$dE_r = dE \cos \alpha = dE \frac{L}{r} = \frac{Ldq}{4\pi \cdot \varepsilon \cdot \varepsilon_0 r^3}.$$

Then

$$dE = \frac{Ldq}{4\pi \cdot \varepsilon \cdot \varepsilon_0 r^3}.$$

Given this, we get:

$$E = \int \frac{Ldq}{4\pi \cdot \varepsilon \cdot \varepsilon_0 r^3} = \frac{L}{4\pi \cdot \varepsilon \cdot \varepsilon_0 r^3} \int dq = \frac{Lq}{4\pi \cdot \varepsilon \cdot \varepsilon_0 r^3}. \quad (1)$$

Given that  $r^2 = R^2 + L^2$ , we obtain the electric field strength on the axis of the ring:

$$E = \frac{Lq}{4\pi \cdot \varepsilon \cdot \varepsilon_0 (R^2 + L^2)^{3/2}}. \quad (2)$$

### Problems for self-solving

3.1. Two point charges, being in the air, at a distance of 20 cm from each other, interact with some force. At what distance do you need to place these charges in the oil to get the same force of interaction?

3.2. Construct a dependency graph of the force of interaction between two point charges on the distance between them in the interval  $2 \leq r \leq 10$  cm every 2 cm. Charges respectively are equal  $2 \cdot 10^{-8}$  Cl and  $3 \cdot 10^{-8}$  Cl.

3.3. How many times is the force of Newtonian attraction between two protons less than the force of their Coulomb repulsion? The charge of the proton is numerically equal to the charge of the electron.

3.4. Two positive point charges  $q$  and  $4q$  are fixed at a distance of 60 cm from each other. Determine at what point on the line passing through the charges, you should place the third charge  $q_1$ , so that it is in equilibrium. Indicate what sign this charge must have in order for the equilibrium to be stable.

3.5. A negative charge is placed in the center of the square, at the vertices of which there are charges of  $2,33 \cdot 10^{-9}$  Cl. Find the magnitude of this charge if the resultant force acting on each charge is zero.

3.6. When bombarding a stationary sodium nucleus with an alpha particle, the repulsive force between them reached 137,2 N. 1) At what smallest distance did the alpha particle approach the nucleus of the sodium atom? 2) What was the speed of the alpha particle? Neglect the influence of the electronic shell of the sodium atom.

3.7. Determine the electric field strength created by a point charge of 10 nCl at a distance of 10 cm from it. Dielectric - oil.

3.8. Determine the electric field strength at a distance of  $2 \cdot 10^{-8}$  cm from the monovalent ion. The charge of the ion is considered to be point.

3.9. Two identical point charges of the same name  $2 \text{ nCl}$  are placed at a distance of  $1 \text{ m}$  from each other. Calculate the electric field strength at a point in the middle of the distance between charges.

3.10. What is the Earth's charge if the electric field strength near the Earth's surface is  $90 \text{ V/m}$ ? Consider the Earth a sphere with a radius of  $6400 \text{ km}$ .

3.11. Two point charges  $q_1 = 2q$  and  $q_2 = -q$  are at a distance  $d$  from each other. Find the position of a point on the line passing through these charges, the field strength  $E$  is zero.

3.12. The electric field is created by two point charges of  $10 \text{ nCl}$  and  $20 \text{ nCl}$ , located at a distance of  $20 \text{ cm}$  from each other. Determine the field strength at a point that is  $30 \text{ cm}$  away from the first charge and  $50 \text{ cm}$  from the second.

3.13. Draw on one graph a curve depending on the intensity of the electric field from the distance in the interval  $1 \leq r \leq 5 \text{ cm}$  through each  $1 \text{ cm}$ , if the field is created: 1) point charge  $33,3 \text{ nCl}$ ; 2) infinitely long charged thread with linear charge density  $1,67 \text{ nCl/cm}$ ; 3) infinitely extended charged plane with a surface charge density of  $2,5 \text{ нКл/см}^2$ .

3.14. Two parallel planes are simultaneously charged with a surface charge density of  $0,5 \text{ μCl/m}^2$  and  $1,5 \text{ μCl/m}^2$ . Determine the field strength: a) between the planes; b) outside the planes.

3.15. In the vertices of a square with a side of  $1 \text{ m}$  are equal charges of the same name. The potential of the field created by them in the center of the square is  $50 \text{ V}$ . Determine the magnitude of the charge.

3.16. Two identical metal charged balls weighing  $0,2 \text{ kg}$  each are at some distance from each other. Find the charge of the balls, if it is known that at this distance their electrostatic energy is a million times greater than their mutual gravitational energy.

3.17. Construct a dependency graph of the potential electrostatic energy of two point charges on the distance between them in the interval  $2 \leq r \leq 10 \text{ cm}$  every  $2 \text{ cm}$ . Charges are equal  $q_1 = 10^{-9} \text{ Cl}$  and  $q_2 = 3 \cdot 10^{-9} \text{ Cl}$ ;  $\epsilon = 1$ . Construct a schedule for the following cases: 1) charges of the same name; 2) charges of different names.

3.18. Determine the potential of a field point located at a distance of  $10 \text{ cm}$  from the center of a charged sphere with a radius of  $1 \text{ cm}$ . Solve the problem under the following conditions: 1) given surface charge density on the ball, equal to  $10^{-11} \text{ Cl/cm}^2$ ; 2) given the potential of the ball, equal to  $300 \text{ V}$ .

3.19. The radius of the charged metal sphere is  $0,1 \text{ m}$ ; its potential is  $300 \text{ V}$ . Determine with what density the charge is distributed over the surface of the sphere.

3.20. The potentials of points A and B are  $0,3 \text{ kV}$  and  $1,2 \text{ kV}$  in an electric field. What work needs to be done in order to move a positive charge of  $30 \text{ nCl}$  from point A to point B?

3.21. The ionization potential of the atom is 16,2 V. What is the lowest velocity of the electron to ionize the atom on impact?

3.22. What work is done when transferring a point charge of 20 nC from infinity to a point located at a distance of 1 cm from the surface of a sphere with a radius of 1 cm with a surface charge density of 1 nC/cm<sup>2</sup>?

3.23. The work of 4 μJ was performed by moving the charge of 20 nC between two points of the field by external forces. Determine the work of field forces and the potential difference of these points of the field.

3.24. What work needs to be done to bring the charges of 5 nC and 2 nC, which were at a distance of 1 m, closer to 0,1 m?

3.25. A point charge of 1 pC moves in a semicircle, in the center of which is a charge of 1 kC. Determine the work of moving the charge.

3.26. What distance will a negatively charged ball of mass 10 g fly in a uniform electric field with a voltage of 200 V/m to a stop if it flies into a field with an initial velocity of 100 m/s in the direction of the lines of force? The charge of the ball is 0,01 C.

3.27. An electron, flying in an electric field from point A to point B, increases its speed from 2000 km/s to 2400 km/s. Determine the difference of potentials of the electric field between points A and B.

3.28. An electron flies from point A to point B, the difference of potentials between which is 182 V. What is the velocity of the electron at point B if it was at rest at point A?

3.29. At what speed do the electrons emitted by the cathode reach the anode of the electron lamp if the voltage between the cathode and the anode is 180 V?

3.30. What should be the electric field strength for an electron to ionize a gas atom with ionization energy of  $2 \cdot 10^{-20}$  J? The free path length of the electron in this gas is 0,5 μm.

3.31. In a field of infinite uniformly charged plane with a surface charge density of 1 μC / m<sup>2</sup>, the charge is moved from a point at a distance of 0.5 m from the plane to a point at a distance of 1 m from it. Determine the charge if 0.1 mJ work is performed.

3.32. Find the velocity of the electron that has passed the difference of potentials U: 1 V, 5 V, 10 V, 100 V, 1000 V.

3.33. An electron in a homogeneous electric field receives an acceleration of  $10^{14}$  cm/s<sup>2</sup> within 1 s. Find: 1) the work of electric field forces during this time; 2) the difference of potentials passed by the electron.

3.34. What work is done by the forces of the field if the charges of the same name 3 nC and 5 nC, which were at a distance of 5 cm, diverged by a distance of 10 cm?

3.35. The charge of 1 nC moved from a point with a potential of 200 V to a point with a potential of 1200 V. Determine the work of field forces.

3.36. What work needs to be done to transfer a point charge of 10 nC from infinity to a point located at a distance of 0,01 m from surface spheres with a radius of 0,01 m with a surface charge density of  $88,5 \text{ nC/cm}^2$ ?

3.37. A ball weighing 40 mg, charged with a positive charge of 1 nC, moves at a speed of 10 cm/s. At what distance can the ball approach the positive point charge of 1,33 nC? Graph the distribution of the potential created by a positive point charge of 1,33 nC in the interval:  $0,1 r_0 \leq r \leq 2 r_0$ .

3.38. How far can two electrons approach if they move towards each other at a relative speed of 108 cm/s? Graph the distribution of the potential energy of interaction of these electrons in the interval:  $0,1 r_0 \leq r \leq 2 r_0$ .

3.39. The proton (the nucleus of the hydrogen atom) moves at a speed of  $7,7 \cdot 10^8 \text{ cm/s}$ . What is the smallest distance  $r_0$  this proton can approach to the nucleus of an aluminum atom? The charge of the nuclei of the aluminum atoms  $q = Ze_0$ , where  $Z$  is the ordinal number of the atom in the table Mendeleev and  $e_0$  is the charge of the proton, numerically equal to the charge of the electron. The mass of the proton is equal to the mass of the hydrogen atom. The proton and core of an aluminum atom are considered point charges. The influence of the electronic shell of the aluminum atom is neglected. Graph the distribution of the potential energy of interaction between proton and aluminum nucleus in the interval:  $0,1 r_0 \leq r \leq 2 r_0$ .

3.40. Two balls with charges of 6,67 nC and 13,33 nC are at a distance of 40 cm. What work should be done to bring them closer to a distance of 25 cm? Graph the distribution of the potential energy of the electrostatic interaction of these two balls in the interval:  $10 \leq r \leq 50 \text{ cm}$ .

3.41. A ball of mass 1 g and a charge of 10 nC moves from point A, the potential of which is equal to 600 V, to point B, the potential of which is zero. Why was its speed equal to point A, if at point B it became equal to 20 cm/s? Graph the distribution of the potential energy of the electrostatic interaction of the ball and its kinetic energy at the interval between points A and B.

3.42. The point charge of 0,67 nC is at a distance of 4 cm from the infinitely long charged filament. The charge moves a distance of 2 cm under the action of the field; while the work is 5  $\mu\text{J}$ . Find the linear charge density of the thread. Graph the distribution of the potential energy of the electrostatic interaction of this point charge in the interval:  $1 \leq r \leq 5 \text{ cm}$ .

3.43. A point charge of 0,67 nC is located near the charged infinite plane. The charge moves a distance of 2 cm under the action of the field; while the work is 5  $\mu\text{J}$ . Find the surface charge density on the plane. Graph the distribution of the potential energy of the electrostatic interaction of this point charge in the interval:  $0 < r \leq 5 \text{ cm}$ .

3.44. The electric field is created by two parallel plates at a distance of 2 cm from each other; the difference of potentials between them is 120 V. What is the

velocity of the electron under the action of the field, passing a distance of 3 mm along the force line? Graph the distribution of the kinetic energy of the electron in the interval:  $0 < r < 2$  cm.

3.45. Eight charged water drops with a radius of 1 mm and a charge  $10^{-10}$  Cl each merge into one large water drop. Find the potential of a big drop. Graph the distribution of the potential of a large water droplet along its radius in the range:  $0 < r \leq 5$  cm.

3.46. Two metal balls, the first with a charge of  $10^{-8}$  Cl and a radius of 3 cm and the second with a radius of 2 cm and a potential of 9000 V, are connected by a wire, the capacity of which can be neglected. Find: 1) the potential of the first ball to the discharge; 2) charge of the second ball to the discharge; 3) the energy of each ball to discharge. Graph the distribution of the potential created by these balls along the line connecting them, for the case when the balls are not connected by a wire, if the distance between the balls is 50 cm.

3.47. Find the charge and potential of the first ball after the discharge and the charge and potential of the second ball after the discharge under the condition of the previous problem. For the case where the balls are detached from the wire, draw graphically the distribution of the potential created by these balls along the line connecting them, if the distance between the balls is 50 cm.

3.48. The charged ball A with a radius of 2 cm is brought into contact with the uncharged ball B, whose radius is 3 cm. After the balls were disconnected, the energy of the B ball was equal to 0.4 J. What charge was on the ball A before they collided? For the case when the balls are disconnected, graph the potential distribution graphically created by these balls along the line connecting them, if the distance between the balls is 50 cm.

3.49. The difference of potentials between the plates of a flat capacitor is 90 V. The area of each plate is  $60 \text{ cm}^2$  and the charge is 1 nCl. At what distance are the plates?

3.50. The capacitor consists of two concentric spheres. The radius of the inner sphere is equal to 10 cm, the outer – 10,2 cm. The gap between the spheres is filled with paraffin. The inner sphere was charged up to 5  $\mu\text{Cl}$ . Determine the difference of potentials between spheres.

3.51. At what connection of three capacitors with a capacity of 2  $\mu\text{F}$ , 1  $\mu\text{F}$  and 3  $\mu\text{F}$ , the battery capacity will be minimal and at what maximum? Determine the value of the containers of the resulting batteries.

3.52. Calculate the capacity of the battery, which consists of three capacitors with a capacity of 4  $\mu\text{F}$  each, in all possible cases of their connection.

3.53. Condenser with paraffin dielectric charged to the potential difference 150 V. Field tension in it is  $6 \cdot 10^4 \text{ V/m}$ , the area of the plates is  $60 \text{ cm}^2$ . Determine the capacitance of the capacitor.

3.54. A flat mica ( $\varepsilon = 7$ ) capacitor with the area of plates of  $5 \text{ cm}^2$  and a distance between them of  $2 \text{ mm}$  is under a voltage of  $300 \text{ V}$ . Determine the capacitance of the capacitor and the force of attraction between its plates.

3.55. Determine how the capacity, energy and volume energy density of the flat air capacitor will change if the gap between the plates is filled with paraffin.

3.56. The energy of the flat air capacitor is  $4 \text{ }\mu\text{J}$ , the difference of potentials between the covers is  $600 \text{ V}$  and the area of the plates is  $100 \text{ cm}^2$ . Determine the distance between the covers, the tension and the volumetric energy density of the capacitor field.

3.57. The covers of a flat air capacitor with an area of  $100 \text{ cm}^2$  and a charge of  $4 \text{ }\mu\text{C}$  are diluted by  $1 \text{ cm}$ . Determine the work done at the same time.

3.58. The difference of potentials between the covers of a  $2 \text{ kV}$  flat capacitor, the distance between them is  $2 \text{ cm}$ , the charge on each cover is  $1 \text{ nC}$ . Determine the force of attraction between the covers and the energy of the capacitor.

3.59. When studying photographic phenomena, a spherical capacitor is used, consisting of a central cathode - a metal ball with a diameter of  $1,5 \text{ cm}$  - and an anode - the inner surface of a spherical flask with a diameter of  $11 \text{ cm}$ . The air is pumped out of the flask. Find the capacity of such a capacitor.

3.60. What will be the potential of the ball with a radius of  $3 \text{ cm}$ , if: 1) give it a charge of  $1 \text{ nC}$ ; 2) surround it with another ball of  $4 \text{ cm}$  radius, concentric to the first and connected to the ground?

3.61. What limits can change the capacity of the system consisting of two capacitors of variable capacity, if the capacity of each of them can vary from  $10 \text{ pF}$  to  $450 \text{ pF}$ ?

3.62. The ball with a radius of  $1 \text{ m}$  is charged to a potential of  $80 \text{ kV}$ . Find the energy of the charged ball.

3.63. The bullet, immersed in kerosene, has a potential of  $4500 \text{ V}$  and a surface charge density of  $1,13 \text{ nC/cm}^2$ . Find: 1) radius; 2) charge; 3) capacity; 4) energy of the sphere.

3.64. The area of the flat air capacitor plates is  $100 \text{ cm}^2$  and the distance between them is  $5 \text{ mm}$ . Find out what potential difference was applied to the plates of the capacitor, if it is known that at the discharge of the capacitor allocated  $4,19 \cdot 10^{-3} \text{ J}$  heat.

3.65. In a flat horizontally located capacitor, a charged droplet of mercury is in equilibrium at an electric field strength of  $600 \text{ V/cm}$ . The charge of the drop is equal to  $8 \cdot 10^{-19} \text{ C}$ . Find the radius  $r_0$  of the drop.

3.66. The flat capacitor can be used as a sensitive microbalance. Inside the horizontally located flat capacitor, the distance between the plates of which is  $3,84 \text{ mm}$ , there is a charged particle of  $4,8 \cdot 10^{-19} \text{ C}$ . In order for the particle to be in equilibrium, it was necessary to apply a potential difference of  $40 \text{ V}$  between the plates of the capacitor. Find the mass  $m_0$  of the particle.

3.67. The electron, having passed in a flat capacitor path from one plate to another, acquires a speed of 108 cm/s. The distance between the plates is 5,3 mm. Find the potential difference  $\Delta\varphi_0$  between the plates.

3.68. Two balls of the same radius 1 cm and weight  $4 \cdot 10^{-5}$  kg are suspended on threads of the same length so that their surfaces touch. When the balls were given a charge  $q_0$ , the threads diverged to some angle and the tension of the threads became equal  $4,9 \cdot 10^{-4}$  N. Find the potential of charged balls, if it is known that the distance from the point of suspension to the center of each ball is 10 cm.

3.69. The electric field is created by an infinite uniformly charged plane with a surface charge density of  $2 \mu\text{Cl}/\text{m}^2$ . In this field along the line that makes up the angle  $\alpha_0 = 60^\circ$  with the plane, from point 1 to point 2, the distance between which is 20 cm, moves point electric charge 10 nCl. Determine the operation of the field forces to move the charge.

3.70. An electron with some initial velocity flies into a flat capacitor parallel to the plates at an equal distance from them. A potential difference of 300 V is applied to the capacitor plates. The distance between the plates is 2 cm, the length of the capacitor is 10 cm. What should be the maximum initial velocity of the electron so that the electron does not fly out of the capacitor? Solve the same problem for the alpha particle. Draw the trajectory of their movement in the capacitor at the appropriate scale for both particles.

3.71. A potential difference of 150 V is applied between the plates of the flat capacitor at a distance of 5 mm from each other. A plane-parallel porcelain plate 3 mm thick is attached to one of the plates. Find the electric field strength in air and porcelain.

3.72. The difference of 100 V potentials is applied between the plates of a flat capacitor at a distance of 1 cm from each other. A flat-parallel plate of crystalline bromous thallium ( $\epsilon = 173$ ) 9,5 mm thick adjacent to one of the plates. After disconnecting the capacitor from the voltage source, the crystal plate is removed. What will be the potential difference between the capacitor plates after that?

3.73. The cylindrical capacitor consists of an inner cylinder with a radius of 3 mm, two layers of insulator and an outer cylinder with a radius of 1 cm. The first layer of insulator 3 mm thick adjacent to the inner cylinder. Find the ratio of the potential drops in these layers.

3.74. Flat air capacitor, the distance between the plates of which is equal to 5 mm, charged to the potential of 6 kV. The area of the capacitor plates is  $12,5 \text{ cm}^2$ . The capacitor plates slide to a distance of 1 cm, and the capacitor remains connected to the voltage source. Find: 1) change capacitance capacitor; 2) change in the voltage flow through the electrode area; 3) change in the volumetric energy density of the electric field.

## SECTION 4. LAWS OF DC CURRENT

### Basic laws and formulas

DC power	$I = \frac{q}{t}$
Electric current density	$j = \frac{I}{S}$
Resistance of a homogeneous conductor	$R = \rho \frac{l}{S}$
Conductor electrical conductivity	$G = \frac{1}{R}$
Specific electrical conductivity of the substance	$\sigma = \frac{1}{\rho}$
Dependence of resistivity of the metal sample on temperature	$\rho = \rho_0(1 + \alpha t)$
Resistance of conductors: consistent connection of conductors:  b) parallel connection of conductors:	$R = \sum_{i=1}^n R_i$ $\frac{1}{R} = \sum_{i=1}^n \frac{1}{R_i}$
Ohm's law for an inhomogeneous section of a circle	$I = \frac{(\varphi_1 - \varphi_2) + \varepsilon_{12}}{R} = \frac{U}{R}$
Ohm's law for a homogeneous section of a circle	$I = \frac{\varphi_1 - \varphi_2}{R} = \frac{U}{R}$
Ohm's law for a closed circle	$I = \frac{\mathcal{E}}{R + r}$
Kirchhoff's first rule	$\sum_{k=1}^n I_k = 0$
Kirchhoff's second rule	$\sum_{k=1}^n I_k R_k = \sum_{i=1}^m \mathcal{E}_i$
Work performed by electric current on a section of a circle	$A = IUt = I^2 R t = \frac{U^2}{R} t$
Current power on a section of a circle	$P = IU = I^2 R = \frac{U^2}{R}$
Joule-Lenz law	$Q = I^2 R t$

Current density	$j = en\langle v \rangle$
Ohm's law in differential form	$j = \sigma E$
Joule-Lenz's law in differential form	$\omega = \sigma E^2$
Thermo-e.r.s., which occurs in the thermocouple	$\varepsilon = \alpha(T_1 - T_2)$
Faraday's first law for electrolysis	$m = kq$
Faraday's second law for electrolysis	$k = \frac{M}{FZ}, F = 96,5 \text{ kCl/mole}$
Mobility of charge carriers	$\mu = \frac{\langle v \rangle}{E}$
Ohm's law in differential form for electrolytes and gases	$j = qn(\mu_+ + \mu_-)E$
Saturation current density at thermoelectronic emission	$j_{\text{nac}} = BT^2 e^{-\frac{A}{kT}}$

### Examples of solving problems

**Problem 1.** Source of current with e.r.s. and internal resistance  $r$  is closed to the rheostat. Express the power of the  $P_1$  that is allocated to the outer part of the circle as a function of the current. At what current power the power will be maximal.

**Given:**  $\varepsilon, r$ .

**Find:**  $P_1 - ?$

Full power current source:

$$P = I\varepsilon.$$

Part of this power  $P_2 = I^2 r$  is released inside, the other in the outer part of the circle:

$$P_1 = I\varepsilon - I^2 r. \quad (1)$$

The graph of this function is a parabola, the branches of which are directed downwards. Convert expression (1):

$$P_1 = r \left( I^2 - 2 \frac{\varepsilon}{2r} I + \frac{\varepsilon^2}{4r^2} - \frac{\varepsilon^2}{4r^2} \right) = -r \left( I - \frac{\varepsilon}{2r} \right)^2 + \frac{\varepsilon^2}{4r}. \quad (2)$$

From here it is visible, coordinates of top of a parabola are in a point:

$$I_1 = \varepsilon / (2r), \quad P_{1m} = \varepsilon^2 / (4r).$$

Thus at the strength of the current

$$I_1 = \varepsilon / (2r) \quad (3)$$

the power allocated in the outer part of the circle will have the maximum value:

$$P_{lm} = \varepsilon^2 / (4r).$$

Let the outer area of the circle has resistance  $R$ , at which the current is equal to  $I_1$ . Then according to Ohm's law for a closed circle:

$$I_1 = \varepsilon / (R + r).$$

Comparing this expression with formula (3) we find that:

$$R = r.$$

Thus useful power (power, which is allocated to the outer area of the circle) is maximum when the internal resistance of the source is equal to the resistance of the outer section of the circle. At the efficiency current sources:

$$\eta = \frac{R}{R + r} = \frac{r}{2r} = 0,5 \text{ or } \eta = 50\%.$$

**Problem 2.** Source of current, e.r.s. which is the internal resistance  $r$ , closed to the external resistance  $R$ . When you change the resistance, the current in the circuit also changes. Find the dependence of the efficiency current source  $\eta$  from the strength of the current  $I$ .

**Given:**  $\varepsilon$ ,  $r$ ,  $R$ ,  $I$ .

**Find:**  $\eta$  - ?

Current source efficiency:

$$\eta = \frac{P_1}{P}, \quad (1)$$

where  $P_1$  is the power allocated on the outer circle (utility power);  $P$  - full power source. Useful power can be expressed as the difference between full power and  $P_2$  power, which is allocated inside the source:

$$P_1 = P - P_2.$$

At current  $I$  and e.r.s.  $\varepsilon$  we will have:

$$P = I\varepsilon,$$

$$P_2 = I^2 r.$$

where  $r$  is the internal resistance of the source.

Then:

$$P_1 = I\varepsilon - I^2 r.$$

Substituting the value of  $P_1$  and  $P_2$  in formula (1), we get:

$$\eta = \frac{I\varepsilon - I^2 r}{I\varepsilon} = 1 - \frac{r}{\varepsilon} I.$$

The graph of the dependence of the efficiency the current source  $\eta$  from the amperage  $I$  is direct. At current  $I_0 = \varepsilon / r$ , ie at short circuit, efficiency the source is equal to zero.

**Problem 3.** Determine the specific resistance of the conductor length of 2 m, if the current density  $10^6 \text{ A/m}^2$  at its ends the 2 V potential difference is maintained. **Given:**  $l = 2 \text{ m}$ ,  $j = 10^6 \text{ A/m}^2$ ,  $U = 2 \text{ V}$ .

**Find:**  $\rho - ?$

According to the definition of current density:

$$j = \frac{I}{S}, \quad (1)$$

where  $I$  is the current strength,  $S$  is the cross-sectional area of the conductor.

According to Ohm's law:

$$U = IR, \quad (2)$$

where  $U$  - the voltage at the ends of the conductor,  $I$  - the current,  $R$  - the resistance of the conductor.

Conductor resistance:

$$R = \rho \frac{l}{S}, \quad (3)$$

where  $\rho$  - the resistivity of the conductor,  $l$  - the length of the conductor,  $S$  - the cross-sectional area of the conductor.

Substituting in formula (1) the expressions for  $I$  and  $R$  from formulas (2) and (3), we get:

$$j = \frac{U}{\rho l},$$

where we find the resistivity of the conductor:

$$\rho = \frac{U}{jl}.$$

Substituting in the last formula these conditions of the problem we get:

$$\rho = 10^{-6} \text{ Ohm} \cdot \text{m}.$$

**Problem 4.** 300 W, 110 V are written on the bulb of the electric lamp. What additional resistance must be connected to this lamp so that it operates in normal mode at a voltage of 127 V?

**Given:**  $U = 110 \text{ V}$ ,  $P = 300 \text{ W}$ ,  $U_1 = 127 \text{ V}$ .

**Find:**  $R_a - ?$

The lamp is designed for power  $P = 300 \text{ W}$  and voltage  $U = 110 \text{ V}$ . Current through the lamp:

$$I = \frac{P}{U}. \quad (1)$$

Lamp resistance:

$$R = \frac{U}{I}. \quad (2)$$

At a voltage of 127 V for the normal operation of the lamp through it should pass the same current. To obtain it, an additional resistance must be turned on to the voltage source in series with the lamp, on which the excess voltage will fall.

The value of the total resistance of the section of the circle, taking into account (1):

$$R_{sum} = \frac{U_1}{I} = \frac{U_1 U}{P}.$$

Knowing the resistance of the lamp (2), determine the value of the additional resistance:

$$R_a = R_{sum} - R = \frac{U_1 U}{P} - \frac{U}{I}.$$

Hence the magnitude of the additional resistance:

$$R_a = 10,4 \text{ Ohm}.$$

### Problems for self-solving

4.1. Determine the current density in the iron conductor length 10m, if the conductor is under voltage 6 V.

4.2. Two groups of three successively connected elements were connected in parallel. e.r.s. each element is equal to 1,2 V, internal resistance 0,2 Ohm. The resulting battery is closed to external resistance 1,5 Ohm. Find the current in the outer circle.

4.3. To the source with e.r.s. 1,5 V connected the coil with resistance 0,1 Ohm. The ammeter showed the current, which is equal to 0,5 A. When another current source was connected to the current source in series with the same e.r.s., the current in the same coil was equal 10,4 A. Determine the internal resistance of the first and second current sources.

4.4. Find the temperature of the tungsten thread of incandescent lamp in working condition, if it is known that the resistance of the thread at the time of turn on at temperature 20°C is 12,6 times less than in working condition.

4.5. Tungsten thread electrolamps have length 18 cm and resistance 190 Ohm at temperature 2400°C. What is the diameter of the thread?

4.6. A coil is wound from copper wire length 120 m and cross-section area 24 mm<sup>2</sup>. Find an increase in the resistance of the coil when heated from 20°C to 70°C.

4.7. Two elements ( $\varepsilon_1 = 1,2 \text{ V}$ ,  $r_1 = 0,1 \text{ Ohm}$ ,  $\varepsilon_2 = 0,9 \text{ V}$ ,  $r_2 = 0,3 \text{ Ohm}$ ) are connected by poles of the same name. Determine the current in the circuit.

4.8. How to connect 12 elements with e.r.s. 1,2 V and internal resistance 0,4 Ohm to get the maximum current in the outer circle, the resistance of which 0,3 Ohm? Determine this current.

4.9. The shunted ammeter measures currents up to 10 A. What is the greatest current strength this ammeter can measure without a shunt if the resistance of the ammeter 0,02 Ohm and the resistance of the shunt 5 mOhm?

4.10. The coil and the ammeter are connected in series and connected to a current source. A voltmeter with a resistance of 1 Ohm is connected to the coil clamps. Readings of ammeter 0,5 A, voltmeter 100 V. Determine the resistance of the coil.

4.11. The lamp, switched on the network voltage 200 V, consumes power 40 W and brightly burns, and the temperature of the thread  $3000^{\circ}\text{C}$ . When the voltage 100 V is switched on the network 100 V, the lamp consumes power 25 W and barely glows, because the temperature of the thread  $1000^{\circ}\text{C}$  it. Find the resistance of the lamp thread at temperature  $0^{\circ}\text{C}$ .

4.12. Electric kettle with a capacity of  $1,25\text{ dm}^3$  has the resistance of the heating element 80%, efficiency and works at 220 V. The initial temperature of the water is  $20^{\circ}\text{C}$ . Determine the current power consumed by the kettle, the current in the heating element and the time during which the water in the kettle boils.

4.13. For what value of the resistance of the outer circle power, which gives the source of current in the outer circle, maximum and what is the current and power?

4.14. Element with an internal resistance of 2 Ohm and e.r.s. 10 V is closed by a 12 Ohm conductor. What amount of heat will be released in the conductor in 6 seconds?

4.15. Three conductors with resistances of 3 Ohm, 4 Ohm, and 5 Ohm are connected in parallel. 20 kJ of heat is released on the first conductor for a certain time. Determine the amount of heat released on the third conductor.

4.16. The winding of the electric boiler has two sections. If only the first section is turned on, the water boils through 15 min, if only the second, then 30 min. After how many minutes will the water boil if both sections turn on consistently?

4.17. With external support 3 Ohm current in the circle is equal to 0,8 A, and with resistance 14 Ohm - current is equal to 0,2 A. Determine the value of the maximum power that can be released in the outer circuit of a given current source.

4.18. The crane lifts up at a constant speed weight of 0,6 t for 40 s to a height of 16 m. The engine operates under 220 V. Find the current consumed by the motor if its efficiency is equal to 80%.

4.19. The electromotive force of the battery is 3 V, the internal resistance is 0,2 Ohms. The outer circuit absorbs power 1,2 W. Determine the amount of external resistance.

4.20. The source of current at short circuit gives the current 2,6 A. If the source is closed to the resistance of the 2,9 Ohm, the current power in the outer circle is 1,6 W. Find the internal resistance of the current source.

4.21. Lamp reostat consists of five electric lamps, included in parallel. Find the resistance of the reostat: 1) when all the lamps are lit; 2) when unscrewed: a) one; b) two; c) three; d) four lamps. The resistance of each lamp is equal to 350 Ohms.

4.22. How many turns of nichrome wire with a diameter of 1 mm should be wound on a porcelain cylinder with a radius of 2,5 cm to get an oven with a resistance of 40 Ohm?

4.23. The copper wire coil has a resistance of 10,8 Ohm. The weight of the copper wire is 3,41 kg. How many meters of wire and what diameter is wound on the coil?

4.24. Find the resistance of an iron rod with a diameter of 1 cm if its mass is 1 kg.

4.25. Two cylindrical conductors, one made of copper and the other made of aluminum, have the same length and the same resistance. How many times is copper wire heavier than aluminum?

4.26. Determine the current density in an iron conductor 10 m long if the wire is under voltage of 6 V.

4.27. Find a drop in potential on a copper wire 500 m long and 2 mm diameter if the current in it is 2 A.

4.28. The voltage at the ends of the conductor with a resistance of 5 Ohm for 0,5 s evenly increases from 0 to 20 V. What charge passes through the conductor during this time?

4.29. Determine the current density, which passes along the resistor length of 5 m, if the difference of potentials is maintained at the ends of it 2 V. Specific resistance of the material  $2 \mu\text{Ohm}\cdot\text{m}$ .

4.30. Element with e.m.f. 1,1 V and an internal resistance of 1 Ohm is closed to an external resistance of 9 Ohm. Find: 1) the strength of the current in the circuit; 2) falling potential on the outer section; 3) falling potential inside the element; 4) with which the efficiency the element is running.

4.31. Construct a graph of the potential drop dependence in the outer circle from the external resistance for the electrical circuit of the previous problem. Take the external resistance within  $0 \leq R \leq 10 \text{ Ohm}$  every 2 Ohm.

4.32. The decrease in voltage in the outer circle is equal to the 5,1 V. Determine the current in the circuit, e.r.p. and efficiency of the current source, if its internal resistance is 1,5 Ohm, and the resistance of the outer circle is 8,5 Ohm.

4.33. Element with e.m.f. 2 V has an internal resistance of 0,5 Ohm. Determine the drop in the potential inside the element at the current force in the circle 0,25 A. Find the external resistance of the circle under these conditions.

4.34. The electromotive force of the element is 1,6 V and its internal resistance is 0,5 Ohm. Why is the efficiency element at a current of 2,4 A?

4.35. The electromotive force of the element is equal to 6 V. At an external resistance of 1,1 Ohm, the current in the circuit is 3 A. Find the potential drop inside the element and its resistance.

4.36. Element, rheostat and ammeter are connected in series. The element has an e.r.s. 2 V and an internal resistance of 0,4 Ohm. The ammeter shows a current of 1 A. With which the efficiency does the element work?

4.37. Two groups of three consecutively connected elements are connected in parallel. E.m.f. each element is 1,2 V, internal resistance 0,2 Ohm. The resulting battery is closed at an external resistance of 1,5 Ohm. Find the current in the outer circle.

4.38. Determine the short circuit current of the battery, e.m.f. which is 15 V, if when connected to it resistance 3 Ohm current in the circuit is 4 A.

4.39. The circle includes a series of copper and steel conductors of equal length and diameter. Find: 1) the ratio of quantities, heat released in these conductors; 2) the ratio of voltage drop across these conductors.

4.40. Solve the previous problem for the case when the conductors are connected in parallel.

4.41. Element, e.m.f. which is equal to 6 V, gives a maximum current of 3 A. Find the largest amount of heat that can be released on the external resistance for 1 min.

4.42. Determine: 1) total power; 2) useful power; 3) battery efficiency, e.m.f. which is 240 V, if the external resistance is 23 Ohm and the battery resistance is 1 Ohm.

4.43. How many watts does the heater of an electric kettle consume if 1 liter of water boils in 5 minutes? What is the resistance of the heater if the voltage is 120 V? The initial water temperature is 20°C. Neglect heat loss.

4.44. The current density in the copper conductor is 3 A/mm<sup>2</sup>. Find the electric field strength in the conductor.

4.45. The current density in the conductor is 1 MA/m<sup>2</sup> at a voltage of 200 V at its ends. Determine the resistivity and material of the conductor, if its length is 500 m.

4.46. Determine the potential difference at the ends of the nichrome conductor with a length of 1 m, if the density of the current flowing through it is 2·10<sup>8</sup> A/m<sup>2</sup>.

4.47. The current density in a copper conductor with a length of 4 m is equal to 1 MA/m<sup>2</sup>. Determine the potential difference at the ends of the conductor.

4.48. The potential difference between two points A and B is 9 V. There are two conductors whose resistances are 5 and 3 Ohm, respectively. Find the amount of heat released in each of the conductors for 1 s, if the conductors between A and B are included: 1) in series; 2) in parallel.

4.49. Two electric lamps are connected to the mains in parallel. The resistance of the first lamp is 360 Ohm; the resistance of the second is 240 Ohm. Which of the lamps absorbs more power? How many times?

4.50. Find the amount of heat released every second per unit volume of copper wire at a current density of 30 A/cm<sup>2</sup>.

4.51. In a copper conductor with a volume of 6 cm<sup>3</sup> when passing a direct current for 1 min, the amount of heat 216 J is released. Calculate the electric field strength in the conductor.

4.52. To heat 4,5 liters of water from 230°C to boiling, the heater consumes 0,5 kWh of electricity. Why is the efficiency heater?

4.53. The current in the metal conductor is 0,8 A, the cross section of the conductor is 4 mm<sup>2</sup>. Assuming that in each cubic centimeter of metal  $2.5 \cdot 10^{22}$  free electrons, determine the average speed of their ordered motion.

4.54. Determine the average velocity of the ordered motion of electrons in a copper conductor at a current of 10, A and a cross section of the conductor equal to 1 mm<sup>2</sup>. Assume that each copper atom has two conduction electrons.

4.55. Determine the power of the electric kettle heater, if you can boil 2 liters of water in 10 minutes, the initial temperature of which is 20°C, efficiency heater is 70%.

4.56. Given three electric lamps, designed for a voltage of 110 V each, power which is equal respectively 40, 40 and 80 W. How do I turn on these three lamps so that they give a normal incandescent at 220 V? Find the current that flows through the lamps at normal incandescent.

4.57. A 120-volt lamp with a power of 40 watts was given. What additional resistance must be connected in series with the lamp so that it gives normal incandescence at a voltage of 220 V? How many meters of nichrome wire with a diameter of 0,3 mm must be taken to get such resistance?

4.58. Through supports 1 Ohm and 2 Ohm, which are connected in parallel current flows 6 A. What power is released on the poles?

4.59. Find the amount of heat released every second per unit volume of copper wire at a current density of 30 A/cm<sup>2</sup>.

4.60. Light bulb and reostat, connected in series, connected to the current source. Voltage on lamp clamps 40 V, resistance reostat 10 Ohm. The outer section of the circle consumes power 120 W. Find the current of the circle.

4.61. On the stove with a capacity of 0,5 kW is a kettle, which is filled with 1 liter of water at a temperature of 16°C. The water in the kettle boiled for 20 minutes after turning on the stove. How much heat is lost by heating the kettle itself, by radiation, etc.?

4.62. How much water can be boiled, spending 3kWh electricity? The initial water temperature is 100°C. Neglect heat loss.

4.63. Element, e.m.f. whose  $\varepsilon$  and the internal resistance  $r$ , closed to the external resistance  $R$ . The maximum power in the outer circuit is 9 W. The current under these conditions in the circuit is 3 A. Find the values of  $\varepsilon$  and  $r$ .

4.64. The winding in an electric pan consists of two identical sections. The resistance of each section is 20 Ohm. After how much time will boil 2,2 liters of water, if: 1) one section is included; 2) both sections are included in series; 3) are both sections included in parallel? The initial water temperature is 16°C, the mains voltage is 110 V and efficiency of heater is 85%.

4.65. The electric kettle has two windings. When you turn on one of them, the water in the kettle will boil in 15 minutes, when you turn on the other - in 30 minutes. After how much time the water will boil in the kettle, if you turn on both windings: 1) in series; 2) in parallel?

4.66. The electric kettle with 600 cm<sup>3</sup> of water at 9°C, the winding resistance of which is equal to 16 Ohm, forgot to turn off. How long after turning on all the water in the kettle will boil? Voltage is 120 V, efficiency kettle is 60%.

4.67. 100 g of mercury evaporates every minute in the mercury diffusion pump. Why should the resistance of the pump heater be equal if the heater is connected to a mains supply of 127 V? The heat of vaporization of mercury is taken to be  $2,96 \cdot 10^5$  J / kg.

4.68. The current in the conductor with a resistance of 100 Ohm increases evenly from 0 to 10 A for 30 s. Determine the amount of heat released during this time in the conductor.

4.69. Nitrogen is ionized by X-rays. Determine the conductivity of nitrogen if 107 pairs of ions are in each cubic centimeter of gas in equilibrium conditions. Motility of positive ions 1,27 cm<sup>2</sup> / (Vs) and negative 1,81 cm<sup>2</sup> / (Vs).

4.70. How many times will the specific thermoelectron emission of tungsten at a temperature of 2400 K change if the temperature of tungsten is increased by 100 K?

4.71. How many times the cathode of torium tungsten at its operating temperature of 1800 K gives a higher specific emission than the cathode of pure tungsten at the same temperature? The emission constant B for pure tungsten shall be considered equal to 60 A/cm<sup>2</sup>·deg<sup>2</sup> and for torium tungsten 3 A/cm<sup>2</sup>·deg<sup>2</sup>.

4.72. The 0,18 Ohm ammeter is designed to measure currents up to 10 A, the scale of which is divided into 100 divisions. 1) What resistance should be taken and how to turn it on so that you can measure the current up to 100 A with this ammeter? 2) How will the price of the ammeter division change?

## SECTION 5. ELECTROMAGNETISM

### Basic laws and formulas

Correlation between induction and magnetic field strength	$B = \mu\mu_0 H$
Ampere force	$F_A = IBl \sin \alpha$
Lorenz force	$F_{\text{л}} = qvB \sin \alpha$
Hall effect potential difference	$\Delta\varphi = \frac{1}{qn_0} \frac{IB}{a}$
Orbital radius of charged particle in magnetic field	$r = \left  \frac{m}{q} \right  \frac{v_{\perp}}{B}$
Period of rotation of a charged particle in a magnetic field	$T = \left  \frac{m}{q} \right  \frac{2\pi}{B}$
Screw trajectory step	$h = v_{\parallel} T$
Mobility of charge carriers	$u = \frac{v}{E}$
Current density	$j = en_0 uE$
Specific electrical conductivity	$\sigma = en_0 u$
Bio-Savart-Laplace Law	$dB = \frac{\mu\mu_0}{4\pi} \frac{Idl \sin \alpha}{r^2}$
Induction of a magnetic field created by a rectilinear conductor with current	$B = \frac{\mu\mu_0}{4\pi} \frac{I}{r_0} (\cos \varphi_1 - \cos \varphi_2)$
Induction of magnetic field created by the endless straight conductor with current	$B = \frac{\mu\mu_0}{2\pi} \frac{I}{r_0}$
Induction of the magnetic field of the circular current in the center of the turn	$B = \frac{\mu\mu_0 I}{2r}$
Induction of the magnetic field of the circular current on the axis of the turn	$B = \frac{\mu\mu_0}{2} \frac{Ir^2}{(r^2 + h^2)^{3/2}}$
Magnetic moment of turn	$P_m = IS$
Magnetic field strength in an ideal toroid and solenoid	$H = nI$

Magnetic field strength on the axis of a real solenoid	$H = \frac{nI}{2}(\cos \alpha_2 - \cos \alpha_1)$
Elementary flux of the magnetic induction vector	$d\Phi_m = BdS \cos \alpha$
Work on moving a conductor with a current in a magnetic field	$dA = I d\Phi_m$
The moment of force acting on a circuit with a current in a magnetic field	$M = P_m B \sin \alpha$
Electromotive force of induction	$\varepsilon_i = -\frac{d\Psi}{dt}, \quad \Psi = N\Phi_m$
Magnetic flux of self-induction	$\Phi_{mc} = IL$
Ideal solenoid inductance	$L = \mu\mu_0 n^2 V$
The dependence of current on time in transients	$I = I_0 e^{-\frac{R}{L}t} + \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$
Electromotive force of mutual induction	$\varepsilon_2 = -\frac{d}{dt}(M_{21}I_1)$
Magnetic field energy	$W_m = \frac{LI^2}{2}$
The amount of electricity that has passed through the conductor when an induction current occurs in it	$dq = -\frac{1}{R}d\Phi_m$

### Examples of problem solving

**Problem 1.** The electron, having passed the accelerating potential difference of 88 kV, flies into a homogeneous magnetic field perpendicular to its induction line. Magnetic field induction is 0,01 T. Determine the radius of the electron trajectory.

**Given:**  $e = 1,6 \cdot 10^{-19} \text{ C}$ ,  $m = 9,1 \cdot 10^{-31} \text{ kg}$ ,  $B = 0,01 \text{ T}$ ,  $U = 88 \cdot 10^6 \text{ V}$ .

**Find:**  $r - ?$

In the magnetic field with induction  $B$ , the Lorentz force acts on an electron moving at a velocity  $\vec{v}$  perpendicular to  $\vec{B}$ :

$$F = evB. \quad (1)$$

It gives the electron centripetal acceleration when it moves in a circle:

$$evB = m \frac{v^2}{r}, \quad (2)$$

where  $m$  is the mass of the electron,  $e$  is its charge,  $r$  is the radius of the electron's trajectory.

Having passed the accelerating potential difference  $U$ , the electron will have kinetic energy  $\frac{mv^2}{2}$ , which is equal to the work of  $A$  of the electric field forces:

$$\frac{mv^2}{2} = eU . \quad (3)$$

From the equality (3) we find the velocity of the electron:

$$v = \sqrt{\frac{2eU}{m}} . \quad (4)$$

From the equalities (2) and (4) we find the radius of the trajectory:

$$r = \frac{1}{B} \sqrt{\frac{2Um}{e}} . \quad r = \frac{1}{10^{-2}} \sqrt{\frac{2 \cdot 88 \cdot 10^3 \cdot 9,1 \cdot 10^{-31}}{1,6 \cdot 10^{-19}}} = 0,1 \text{ m}.$$

**Problem 2.** The solenoid 20 cm long and 4 cm diameter has a dense three-layer winding of a conductor with a diameter of 0,1 mm. A current of 0,1 A passes through the solenoid winding. A voltage of 3000 A/m is equal to an induction of 1,7 T. Find magnetic permeability, solenoid inductance, energy and volumetric density of solenoid field energy.

**Given:**  $l = 0,2 \text{ m}$ ,  $D = 0,04 \text{ m}$ ,  $N = 3$ ,  $d = 10^{-4} \text{ m}$ ,  $I = 0,1 \text{ A}$ .

**Find:**  $\mu$ ,  $L$ ,  $W$ ,  $\omega$  - ?

The field inside the solenoid can be considered homogeneous. Then the field strength:

$$H = In,$$

where  $I$  is the current strength in the winding,  $n$  is the number of turns per unit length of the solenoid,  $N$  is the number of winding layers,  $d$  is the diameter of the conductor.

Then:

$$H = \frac{IN}{d}; H = \frac{0,1 \text{ A} \cdot 3}{10^{-4}} = 3000 \text{ A/m}.$$

From the graph  $B = f(H)$  we find that the intensity of 3000 A/m corresponds to an induction of 1,7 T. Using the relationship between induction and intensity:

$$B = \mu_0 \mu H$$

determine the magnetic permeability:

$$\mu = \frac{B}{\mu_0 H},$$

$$\mu = \frac{1,7}{12,56 \cdot 10^{-7} \cdot 3000} = 450.$$

The inductance of a solenoid is determined by the ratio:

$$L = \mu_0 \mu n^2 l S$$

where  $l$  is the length,  $S$  is the cross-sectional area of the solenoid.

$$L = \mu_0 \mu \frac{N^2}{d^2} l \frac{\pi D^2}{4}. \text{ Then: } L = \frac{12,56 \cdot 10^{-7} \cdot 450 \cdot 3^2 \cdot 0,2 \cdot 3,14 \cdot 4^2 \cdot 10^{-4}}{4 \cdot 1 \cdot 10^{-8}} = 128 \text{ Hn.}$$

Volumetric density of magnetic field energy:

$$\omega = \frac{BH}{2},$$

$$\omega = \frac{1,7 \cdot 3000}{2} = 2,55 \cdot 10^3 \text{ J./m}^3.$$

The magnetic field energy of the solenoid:

$$W = \frac{LI^2}{2},$$

$$W = \frac{128 \cdot 1 \cdot 10^{-2}}{2} = 0,64 \text{ J.}$$

**Problem 3.** A turn with a radius of 5 cm with a current of 1 A is placed in a uniform magnetic field with a strength of 5000 A/m so that the normal to the turn forms an angle of  $60^\circ$  with the direction of the field. What kind of work do the field forces do when the turn is returned to a stable position?

**Given:**  $r = 0,05 \text{ m}$ ,  $I = 1 \text{ A}$ ,  $H = 5000 \text{ A/m}$ ,  $\alpha = 60^\circ$ .

**Find:**  $A$  - ?

Work  $A$  when turning a turn with current  $I$  in a magnetic field is determined by:

$$A = I \cdot \Delta\Phi.$$

Here  $\Delta\Phi = \Phi_2 - \Phi_1$  is the change in the magnetic flux through the plane of the turn  $S = \pi r^2$ ;  $\Phi_1 = BS \cos \alpha$  - the magnetic flux penetrating the turn in the initial position, where  $\alpha$  is the angle between the normal  $\vec{n}$  and induction vectors  $\vec{B}$ .

The stable position of the turn in the magnetic circle is the position in which the direction of the normal to it coincides with the direction of the induction vector. Then  $\cos \alpha = 1$ . So,  $\Phi_2 = BS$  and then:

$$\Delta\Phi = B\pi r^2 (1 - \cos \alpha).$$

Because:

$$B = \mu_0 \mu H ,$$

therefore:

$$\Delta\Phi = \mu_0 \mu H \pi r^2 (1 - \cos \alpha) .$$

Substituting the expression for  $\Delta\Phi$  in the expression for  $A$  , we get:

$$A = I \mu_0 \mu H \pi r^2 (1 - \cos \alpha)$$

$$A = 1 \cdot 12,56 \cdot 10^{-7} \cdot 1 \cdot 5 \cdot 10^3 \cdot 3,14 \cdot 25 \cdot 10^{-4} (1 - 0,5) = 2,46 \cdot 10^{-5} \text{ J} .$$

### Problems for self-solving

5.1. Two infinitely long straight parallel conductors in opposite directions flow currents with a force of 50 A and 100 A. The distance between the conductors is 20 cm. Determine magnetic induction at a point spaced 25 cm from the first and 40 cm from the second conductor.

5.2. The magnetic field strength in the center of the circle turn with a radius of 8 cm is 30 A/m. Determine the intensity on the axis of the turn at a point located at a distance of 6 cm from the center of the turn.

5.3. On two infinitely long straight conductors, crossed at a right angle, currents of 30 A and 40 A flow. The distance between the conductors is 20 cm. Determine the magnetic induction at a point that is equidistant from both conductors at a distance of 20 cm.

5.4. An electron in an undeveloped hydrogen atom moves around the nucleus in a circle with a radius of 53 pm. Calculate the strength of the equivalent circular current and the strength of the magnetic field in the center of the circle.

5.5. At a distance of 10 nm from the trajectory of an electron moving rectilinearly, the maximum value of magnetic induction is 160  $\mu\text{T}$ . Determine the speed of an electron.

5.6. The coil 20 cm long has 100 turns. A current of 5 A flows through the winding of the coil. The coil diameter is 20 cm. Determine the magnetic induction at point A, which lies on the axis of the coil at a distance of 10 cm from its end.

5.7. A current of 8 A flows from top to bottom along a long vertical wire. At what distance is the point at which the strength of the magnetic field, which is the vector sum of the strengths of the Earth's magnetic field and the current field, is directed vertically upward? The horizontal component of the Earth's magnetic field is 16 A/m.

5.8. Two circular turns are arranged in mutually perpendicular planes so that their centers coincide. Radius of each turn 2 cm, currents in turns 5 A. Find the strength of the magnetic field in the center of these turns.

5.9. What is the resistance of a conductor 3 cm long, if a force of 45 N acts on it in a magnetic field with an induction of 0,3 T, and the voltage applied to this

conductor is 20 V? The conductor is placed perpendicular to the lines of magnetic induction, the field is uniform.

5.10. In a uniform horizontal magnetic field in a horizontal plane there is a rectilinear aluminum conductor with a current of 10 A. The angle between the conductor and the power lines is  $30^\circ$ . Determine the induction of the magnetic field, considering that the cross-sectional radius of the conductor is 2 mm. Aluminum density  $2,6 \cdot 10^3 \text{ kg/m}^3$ .

5.11. Magnetic field strength in the center of the circular turn is 159 A/m, current in the turn 32 A. Determine the magnetic moment of the turn.

5.12. Determine the mechanical displacement power of a rectilinear conductor 20 cm long at a speed of 5 m/s in a homogeneous magnetic field, the induction of which is 0,1 T. The angle between the vectors  $\vec{B}$  and  $\vec{v}$  is  $\pi/2$ , and the current in the conductor is 50 A.

5.13. Two parallel conductors with a length of 1 m each flow currents of the same force. The distance between the conductors is 1 cm. Currents interact with a force of 1 mN. Find the current strength in the conductors.

5.14. The speed of the aircraft flying horizontally is 900 km/h. Find e.m.f. induction that occurs at the ends of the wings of this aircraft if the vertical component of the induction of the Earth's magnetic field is  $0,5 \cdot 10^{-4} \text{ T}$ , and the wingspan of the aircraft is 12,5 m.

5.15. The wire coil with a radius of 5 cm is in a uniform magnetic field with strength of 2 kA/m. The plane of the turn forms an angle of  $60^\circ$  with a strength vector. Current flows along the turn 4 A. Find the mechanical moment acting on the turn.

5.16. The frame of the galvanometer with 200 turns has dimensions of  $4 \times 1,5 \text{ cm}^2$ . What magnetic moment occurs in the frame when a current of 1 A passes through it?

5.17. A 10 cm long straight conductor through which current 20 A flows is in a uniform magnetic field with an induction of 0,01 T. Find the angle between the directions of the induction vector and the current if a force of 10 mN acts on the conductor.

5.18. A wire turn with a diameter of 20 cm can rotate around a vertical axis that coincides with one of the diameters of the coil. The turn is located in the plane of the magnetic meridian. A current of 10 A was passed through it. Find the mechanical moment that needs to be applied to the turn so that it returns to the initial position, which is part of the magnetic induction of the Earth's field of  $20 \mu\text{T}$ .

5.19. Two thin wires, bent in the form of a ring with a radius of 10 cm, flow the same currents with a force of 10 A in each. Find the force of interaction of these rings if the planes in which the rings lie are parallel, and the distance between the centers of the rings is 1 mm.

5.20.  $\alpha$  – particle accelerated by the 1000 V potential difference flew into a uniform magnetic field directed perpendicular to the particle velocity. Determine the magnetic field strength, knowing that the momentum of the particle is  $10^{21} \text{ kg}\cdot\text{m}^2/\text{s}$ .

5.21. In a vacuum in a magnetic field with induction of  $5\cdot 10^2 \text{ T}$ , protons move along the arc of the circle with a radius of 50 cm. What accelerating potential difference did they pass before that?

5.22. The charged particle passed through the accelerating potential difference of  $10^4 \text{ V}$  and flew into the crossed electric 10 kV/m and magnetic 0,1 T fields at right angles. Find the specific charge of a particle (the ratio of charge to mass) if, moving perpendicular to both fields, the particle does not deviate from a straight path.

5.23. An electron flies into a uniform magnetic field perpendicular to the field lines. The speed of the electron is  $4\cdot 10^7 \text{ m/s}$ . Magnetic field induction  $10^3 \text{ T}$ . What is the normal acceleration of an electron in a magnetic field?

5.24. A particle with an energy of 16 MeV, moves in a homogeneous magnetic field with an induction of 2,4 T in a circle with a radius of 24.5 cm. Determine the charge of this particle if its speed is  $2,72\cdot 10^3 \text{ m/s}$ .

5.25. An electron moves in a circle in a uniform magnetic field at a speed of  $0,8c$  ( $c$  is the speed of light in a vacuum). Magnetic field induction 0,01 T. Determine the radius of the circle in two cases: 1) not taking into account the dependence of the momentum on the speed; 2) taking into account this dependence.

5.26. An electron having a kinetic energy of 1,5 MeV moves in a uniform magnetic field in a circle. Magnetic induction of the field 0,02 T. Determine the period of rotation, taking into account the dependence of the particle's momentum on its speed.

5.27. What should be the velocity of the electron so that its trajectory is straight when moving in mutually perpendicular magnetic and electric fields? The fields are uniform and have strengths of 100 A/m and 500 V/m, respectively.

5.28. The particle carrying the elementary charge flew into a magnetic field with an induction of 0,5 T. Determine the momentum of a particle moving in a magnetic field if its trajectory is a circle arc with a radius of 0,1 cm.

5.29. In a homogeneous magnetic field with a strength of 500 A/m there is a flat round frame with a radius of 10 cm. The angle between the normal to the plane of the frame and the direction of the magnetic field is  $60^\circ$ . Determine the magnetic flux through this plane if  $\mu = 1$ .

5.30. The 1 m rod rotates in a uniform magnetic field about an axis passing through the end of the rod and parallel to the magnetic induction lines. Find magnetic field induction if in one turn the rod crosses the flow of  $2\pi \text{ Vb}$ .

5.31. The circulation of the magnetic field vector along the circuit covering the four-core underwater cable is equal to 15 A. Find the magnitude and direction

of the current in the fourth core, if it is known that the currents of the same direction of 5 A flow through the first two cores, and a current of 3 A flows through the third, the direction of which is opposite to the direction of the first two.

5.32. The flux of the magnetic induction vector through the solenoid (without a core) is  $5 \mu\text{Vb}$ . Find the magnetic moment of this solenoid if its length is 25 cm.

5.33. Calculate the circulation of the magnetic induction vector along a circuit that includes currents of 10 A and 15 A flowing in one direction, as well as a current of 20 A flowing in the opposite direction.

5.34. A solenoid with a length of 1 m and a cross-sectional area of  $16 \text{ cm}^2$  has 2000 turns. Determine the flux linkage if the current in the winding is 10 A.

5.35. The iron core is in a uniform magnetic field with a strength of 1 kA/m. Determine the induction of the magnetic field in the precipitate and the magnetic permeability of the iron.

5.36. The iron ring is wound into one layer of 500 conductor turns. The average diameter of the ring is 25 cm. Determine the magnetic induction in iron and the magnetic permeability of iron, if the current in the winding: 1)  $I = 0,5 \text{ A}$ ; 2)  $I = 2,5 \text{ A}$ .

5.37. The flat circuit, which has an area of  $25 \text{ cm}^2$ , is in a homogeneous magnetic field with an induction of 0,04 T. Determine the magnetic flux penetrating the loop if its plane is located at an angle of  $30^\circ$  to the induction lines.

5.38. A closed solenoid (toroid) with a steel core has 10 turns for each centimeter of length. A current of 2 A flows through the winding of the solenoid. Calculate the magnetic flux in the precipitate if its section is  $4 \text{ cm}^2$ .

5.39. The flat circuit with current 5 A is freely located in a homogeneous magnetic field with induction of 0,4 T; the area of the circuit is  $200 \text{ cm}^2$ . The contour was rotated by an angle of  $40^\circ$  about the axis lying in the plane of the contour. Determine the work that was done if the current in the circuit was kept unchanged.

5.40. The conductor 20 cm long is moved in a uniform magnetic field with an induction of 0,1 T so that its axis is an angle of  $30^\circ$  with a field direction. How do you need to move the conductor so that the potential difference at its ends increases evenly by 1 V in 1 sec?

5.41. A flat circuit with an area of  $100 \text{ cm}^2$  is perpendicular to the induction lines in a uniform magnetic field. By maintaining a constant current of 50 A in the circuit, it was moved to a region of space where the field is absent. Determine the induction of a uniform magnetic field, if 0, J of work was done during the movement of the circuit.

5.42. A frame of 1000 turns, with an area of  $100 \text{ cm}^2$ , rotates uniformly at a frequency of  $10 \text{ s}^{-1}$  in a magnetic field with a strength of  $10^4 \text{ A/m}$ . The axis of rotation lies in the plane of the frame and is perpendicular to the lines of tension. Determine the maximum e.m.f. induction occurring in the frame.

5.43. In a uniform magnetic field, the induction of which is 0,5 T, a conductor with a length of 10 cm moves uniformly. A current of 2 A flows through the conductor. The speed of movement of the conductor is 20 cm/s directed perpendicular to the induction vector. Find the work of moving the conductor in 10 s.

5.44. A circular frame with a radius of 10 cm is placed in a uniform magnetic field with an intensity of 500 A/m. The angle between the normal to the plane of the frame and the direction of the magnetic field is  $60^\circ$ . Determine the magnetic flux through this frame if  $\mu = 1$ .

5.45. How many ampere-turns per meter of length does a closed solenoid (toroid) with a cross-sectional area of  $10 \text{ cm}^2$  with an iron core ( $\mu = 1400$ ) have, if the magnetic flux through it is  $5 \cdot 10^4 \text{ Vb}$ ?

5.46. The coil is freely placed in a homogeneous magnetic field with induction of 20 mT. DC 60 A is maintained in it. The diameter of the turn is 10 cm. What work do you need to do to turn the turn relative to the axis, which coincides with the diameter, by an angle of  $60^\circ$ ?

5.47. A rod with a length of 10 cm, perpendicular to the lines of field induction, rotates in a uniform magnetic field with an induction of 0,4 T in the plane. The axis of rotation passes through one of its ends. Determine the potential difference at the ends of the rod at a rotation frequency of  $16 \text{ s}^{-1}$ .

5.48. A Wire turn with a radius of 4 cm, which has a resistance of 0,01 Ohm, is in a uniform magnetic field with an induction of 0,04 T. The plane of the frame makes an angle of  $30^\circ$  with the field induction lines. How much electricity will flow through the turn if the magnetic field disappears?

5.49. A 10 cm long straight conductor is placed in a uniform magnetic field with 1T induction. Its ends are closed with a flexible wire located outside the field. The resistance of this circle is 0,4 Ohms. What power is required to move the conductor perpendicular to the induction lines at a speed of 20 m/s?

5.50. A current of 0,6 A flows through the winding of the coil, the inductance of which is equal to 0,03 mH. After opening the circuit, the current decreases to almost zero in 120  $\mu\text{s}$ . Determine the average e.m.f. self-induction occurring in the circuit.

5.51. The inductance of the coil is 2 mH. The current flowing through the coil changes according to a sinusoidal law with a frequency of 50 Hz. Determine the average e.m.f. of self-induction, which occurs during the time interval during which the current in the coil changes from the minimum to the maximum value. The amplitude value of the current is 10 A.

5.52. A solenoid with a cross-sectional area of  $5 \text{ cm}^2$  contains 1200 turns. The induction of the magnetic field inside the solenoid at a current of 2 A is equal to 0,01 T. Determine the inductance of the solenoid.

5.53. The solenoid contains 1000 turns. The cross-sectional area of the core is  $10 \text{ cm}^2$ . A current flows through the winding, forming a field with an induction

of 1,5 T. Find the average e.r.s. induction occurring in the solenoid if the current is reduced to zero in 500  $\mu$ s.

5.54. The force of interaction between two parallel infinitely long conductors, through which currents with a force of 1 A pass, is equal to 0,1 N per 1 m of their length. What is the distance between the conductors?

5.55. A straight conductor 10 cm long, through which a current of 10 A flows, is in a magnetic field with an induction of 1 T perpendicular to the lines of induction. What force acts on the conductor?

5.56. A straight conductor with a current is placed in a uniform magnetic field with an induction of 0,2 T. Determine the force acting on the conductor if the length of the conductor is 10 cm, the current is 3 A, and the direction of the current makes an angle of  $30^\circ$  with the direction of the field.

5.57. A straight conductor through which a current of 1000 A flows is placed between the poles of an electromagnet perpendicular to the power lines. With what force does the field act per unit length of the conductor? The induction of the magnetic field is equal to 1 T.

5.58. A straight conductor 10 cm long, through which a current of 20 A flows, is in a uniform magnetic field with an induction of 0,01 T. What is the angle between the direction of the field and the direction of the current, if a force of  $10^{-2}$  N acts on the conductor?

5.59. A conductor with a mass of 1 g and a length of 7,8 cm is in equilibrium in a horizontal magnetic field with strength of  $10^5$  A/m. Determine the current in the conductor if it is perpendicular to the field induction lines and is in a free state.

5.60. A uniform magnetic field with a strength of 225 A/m acts on a 50 cm long conductor embedded in it with a force of  $10^{-4}$  N. What is the current in the conductor if the angle between the current direction and the magnetic field induction vector is  $45^\circ$ ?

5.61. An electron moves in a circle in a uniform magnetic field with strength of  $10^5$  A/m. Calculate the period of rotation of the electron.

5.62. A doubly ionized helium atom ( $\alpha$  - particle) moves in a uniform magnetic field of  $10^5$  A/m in a circle with a radius of 10 cm. Find the speed of  $\alpha$  - particle.

5.63. Determine the Lorentz force acting on an electron that has flown into a magnetic field at an angle of  $30^\circ$ . The field induction is equal to 0,2 T, the electron speed is  $4 \cdot 10^6$  m/s.

5.64. An electron moves in a uniform magnetic field with an induction of 0,02 T in a circle with a radius of 1 cm. Determine the kinetic energy of the electron in joules and electron volts.

5.65. An electron with an energy of 300 eV moves perpendicular to the induction lines of a uniform magnetic field with a strength of 465 A/m. Determine the Lorentz force, speed and radius of the electron trajectory.

5.66. The moment of momentum of a proton in a uniform magnetic field with a strength of 20 kA/m is equal to  $6.6 \cdot 10^{-23}$  kg·m<sup>2</sup>/s. Determine the kinetic energy of a proton if it moves perpendicular to the field induction lines.

5.67. A proton moves in a magnetic field with a strength of  $10^5$  A/m in a circle with a radius of 2 cm. Determine the kinetic energy of the proton.

5.68. A proton that passed through an accelerating potential difference of 600 V flew into a uniform magnetic field with an induction of 0,3 T and began to move in a circle. Calculate the radius of the proton trajectory.

5.69. A charged particle with a speed of  $2 \cdot 10^6$  m/s flew into a uniform magnetic field of 0,52 T. Find the ratio of the charge of a particle to its mass, if the particle described an arc with a radius of 4 cm.

5.70. An electron and a proton, accelerated by the same potential difference, fall into a uniform magnetic field. Compare the radii of curvature of the proton and electron trajectories. The mass of a proton is 1840 times greater than the mass of an electron.

5.71. At a distance of 3 mm, an electron with a kinetic energy of 500 eV moves parallel to a straight long conductor. What force will act on the electron if a current of 10 A is passed through the conductor?

5.72. An electron moves in a uniform magnetic field with an induction of 0,1 T perpendicular to the power lines. Determine the force acting on the electron from the side of the field, if the radius of curvature of the trajectory is 0,5 m.

5.73. Determine the radius of the arc of a circle along which a proton moves in a magnetic field with an induction of  $1,5 \cdot 10^{-2}$  T. The speed of a proton is  $2 \cdot 10^6$  m/s.

5.74. A particle carrying one elementary charge flew into a uniform magnetic field with an induction of 0,5 T. Determine the moment of momentum of a particle in a magnetic field if its trajectory is an arc with a radius of 0,2 m.

5.75. A charged particle, moving in a magnetic field along an arc with a radius of 2 cm, passed through a lead plate. Due to the loss of energy by the particle, the radius of the particle's trajectory became 1 cm. Determine the relative change in the particle's energy.

## SECTION 6. OSCILLATIONS AND WAVES

### Basic laws and formulas

Dependence of the coordinate on time in harmonic oscillations	$x = A \sin(\omega t + \varphi_0)$
Cyclic frequency	$\omega = 2\pi\nu = \frac{2\pi}{T}$
Period of oscillations	$T = \frac{2\pi}{\omega}$

Oscillatory point speed	$v = \frac{dx}{dt} = A\omega \cos(\omega t + \varphi_0)$
Oscillating point acceleration	$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \varphi_0)$
Relationship between force and acceleration for free harmonic oscillations of a material point	$F_x = ma_x = -mA\omega_0^2 \cos(\omega_0 t + \phi_0) = -m\omega_0^2 x = -kx$
Kinetic energy of a material point performing harmonic oscillations	$E_k = \frac{mv^2}{2} = \frac{1}{2}mA^2\omega^2 \cos^2(\omega t + \varphi_0)$
Potential energy of a material point performing harmonic oscillations	$E_n = \frac{kx^2}{2} = \frac{1}{2}kA^2 \sin^2(\omega t + \varphi_0)$
Total mechanical energy of the oscillating point	$E = E_k + E_n = \frac{1}{2}m\omega_0^2 A^2$
Oscillation period of the mathematical pendulum	$T = 2\pi\sqrt{\frac{l}{g}}$
Oscillation period of a spring pendulum	$T = 2\pi\sqrt{\frac{m}{k}}$
Oscillation period of the physical pendulum	$T = 2\pi\sqrt{\frac{I}{mgl}}$
Amplitude of the resulting oscillation, which is obtained by adding harmonic oscillations of one direction	$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\varphi_2 - \varphi_1)$
Initial phase of the resulting oscillation, which is obtained by adding harmonic oscillations of one direction	$\operatorname{tg} \varphi = \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}$
Trajectory of the material point involved in two mutually perpendicular oscillations	$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - 2\frac{x}{A_1}\frac{y}{A_2} \cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$
Dependence of the coordinate on time in decaying oscillations	$x = A_0 e^{-\beta t} \sin(\omega t + \varphi_0)$
Period of natural electromagnetic oscillations that occur in the circuit	$T = 2\pi\sqrt{LC}$
Traveling wave equation	$S = A \sin(\omega t - kx + \varphi_0)$

Elastic wave kinetic energy	$\Delta W_{\kappa} = \frac{1}{2} \rho A^2 \omega^2 \Delta V \cos^2 (\omega t - kx + \varphi_0)$
Elastic wave potential energy	$\Delta W_{\Pi} = \frac{1}{2} \rho A^2 \omega^2 \Delta V \cos^2 (\omega t - kx + \varphi_0)$
Full energy of an elastic wave	$\Delta W = \rho A^2 \omega^2 \Delta V \cos^2 (\omega t - kx + \varphi_0)$
Electromagnetic wave speed	$v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon \mu_0 \mu}}$
Equation of oscillations of vector $\vec{E}$ of an electromagnetic wave	$E = E_0 \sin(\omega t - kx + \varphi_0)$
Equation of oscillations of vector $\vec{H}$ of an electromagnetic wave	$H = H_0 \sin(\omega t - kx + \varphi_0)$
Electromagnetic energy flow vector	$\vec{P} = [\vec{E}\vec{H}]$

### Examples of problem solving

**Problem 1.** Camerton oscillates with frequency  $\nu_0 = 800$  Hz and amplitude  $A = 4$  mm. Find the maximum acceleration of its oscillating branch.

**Given:**  $\nu_0 = 800$  Hz,  $A = 4$  mm.

**Find:**  $a_{\max} - ?$

The tuning fork branch motion equation has the form (in SI system):

$$x = A \sin(2\pi\nu_0 t + \varphi) = 0,004 \sin(2\pi 800 t + \varphi). \quad (1)$$

The initial phase is not known to us, but its value can be arbitrary. By formula for acceleration:

$$a = \frac{dx}{dt} = -(1600\pi)^2 0,004 \sin(1600t + \varphi). \quad (2)$$

The maximum acceleration value corresponds to the moments in time when the sine value included in formula (2) is  $+1$  or  $-1$ , since the absolute acceleration value is significant.

In this case, in terms of absolute value:

$$a_{\max} = (1600\pi)^2 0,004 \approx 10^5 \text{ m/s}^2.$$

**Problem 2.** The amplitude of the pendulum oscillations decreased by three times in  $\Delta t = 8$  min. Determine the attenuation coefficient  $\beta$ .

**Given:**  $\Delta t = 8 \text{ s}$ ,  $\frac{A(t)}{A(t + \Delta t)} = 3$ .

**Find:**  $\beta - ?$

The dependence of the amplitude of attenuating oscillations on time is given by the ratio:

$$A(t) = A_0 e^{-\beta t}.$$

Amplitude ratio over time  $\Delta t$ :

$$\frac{A(t)}{A(t + \Delta t)} = \frac{A_0 e^{-\beta t}}{A_0 e^{-\beta(t + \Delta t)}} = \frac{1}{e^{-\beta \Delta t}} = e^{\beta \Delta t} = 3.$$

Coefficient  $\beta$  can be found by prologarithming the last equality:

$$\beta \Delta t = \ln 3;$$

$$\beta = \frac{\ln 3}{\Delta t}.$$

Calculate:  $\beta = 0,0046 \text{ s}^{-1}$ .

**Problem 3.** A transverse wave propagates along an elastic cord at a speed  $v = 15 \text{ m/s}$ . The period of oscillations of the points of the cord  $T = 1,2 \text{ s}$ , amplitude  $A = 2 \text{ m}$ . Determine a) wavelength  $\lambda$ ; b) phase of oscillations, displacement and speed of a point of the environment, which is located at a distance  $x = 45 \text{ m}$  from wave source at time  $t = 4 \text{ s}$ ; c) the phase difference of oscillations of the two points, which lie on the beam and are distant from the source of waves on  $x_1 = 20 \text{ m}$  and  $x_2 = 30 \text{ m}$ .

**Given:**  $v = 15 \text{ m/s}$ ,  $T = 1,2 \text{ s}$ ,  $A = 2 \text{ m}$ ,  $x = 45 \text{ m}$ ,  $t = 4 \text{ s}$ ,  $x_1 = 20 \text{ m}$ ,  $x_2 = 30 \text{ m}$ .

**Find:**  $\lambda$ ,  $\varphi$ ,  $\Delta \varphi - ?$

The wavelength is equal to the distance the wave travels in one period and can be found from the ratio:

$$\lambda = vT.$$

Substituting the value of quantities  $v$  and  $T$  we get  $\lambda = 18 \text{ m}$ .

Write the wave equation:

$$S = A \cos \omega(t - x/v),$$

Where  $S$  - displacement of the oscillating point,  $x$  - distance of the point from the source of waves,  $v$  - wave propagation speed.

The phase of oscillations of a point with a coordinate  $x$  at a moment in time  $t$  is determined by the expression under the cosine sign:

$$\varphi = \omega \left( t - \frac{x}{v} \right), \text{ also } \varphi = \frac{2\pi}{T} \left( t - \frac{x}{v} \right),$$

where it is taken into account that  $\omega = \frac{2\pi}{T}$ .

After calculating according to the last formula, we get:

$$\varphi = 5,24 \text{ rad, also } \varphi = 300^\circ.$$

The displacement is determined by substituting the amplitude and phase values into the plane wave equation:

$$S = 0,01 \text{ m.}$$

We find the speed of the point by taking the first derivative of the displacement over time:

$$v = \frac{dS}{dt} = -A\omega \sin \omega \left( t - \frac{x}{v} \right) = -\frac{2\pi A}{T} \sin \omega \left( t - \frac{x}{v} \right) = \frac{2\pi A}{T} \sin \varphi.$$

Substituting the value of the quantities  $\pi, A, T, \varphi$  and after calculating, we get:  $v = 0,09 \text{ m/s}$ .

The difference in the phases of oscillations of two wave points is related to the distance  $\Delta x$  between these points by the ratio:

$$\Delta \varphi = \left( \frac{2\pi}{\lambda} \right) \Delta x = \left( \frac{2\pi}{\lambda} \right) (x_2 - x_1).$$

Substituting value of quantities  $\lambda, x_1$  and  $x_2$  and after calculating, we get:  $\Delta \varphi = 3,49 \text{ rad, or } \Delta \varphi = 200^\circ$ .

### Problems for self-solving

6.1. Write the equation of harmonic oscillatory motion with an amplitude of 5 cm, if 150 oscillations occur in 1 minute and the initial phase of the oscillations is  $45^\circ$ .

6.2. Write the equation of motion resulting from the addition of two equally directed harmonic oscillatory motions with the same period of 8 s and the same amplitude of 0,02 m. The phase difference between these oscillations is equal to  $\frac{\pi}{4}$ . The initial phase of one of these oscillations is zero.

6.3. Write the equation of harmonic oscillatory motion, if the initial phase of the oscillations is equal to: 1) 0; 2)  $\pi/2$ ; 3)  $\pi$ ; 4)  $3/2\pi$ ; 5)  $2\pi$ . The amplitude of oscillations is 5 cm and the period of oscillations is 8 s.

6.4. Write the equation of harmonic oscillatory motion with amplitude of 0,1 m, a period of 4 s and an initial phase equal to zero.

6.5. After what time from the beginning of the movement, the points performing the harmonic oscillation will shift from the equilibrium position by half the amplitude? The oscillation period is 24 s, the initial phase is zero.

6.6. 1) Find the amplitude and initial phase of the harmonic oscillation obtained from the addition of equally directed oscillations given by the equations:  $x_1 = 4 \sin \pi t$  cm and  $x_2 = 3 \sin(\pi t + \frac{\pi}{2})$  cm. 2) Write the equation of the resulting oscillation. 3) Provide a vector diagram of the addition of amplitudes.

6.7. Two harmonic oscillations are given:  $x_1 = 3 \sin 4\pi t$  cm and  $x_2 = 6 \sin 10\pi t$  cm. Plot the graphs of these oscillations. Having drawn these oscillations graphically, plot the graph of the resulting oscillation.

6.8. A material point with a mass of 10 g oscillates according to the law  $x = 5 \sin(\frac{\pi t}{5} + \frac{\pi}{4})$  cm. Find the maximum force acting on the point and the total energy of the oscillating point.

6.9. The point participates simultaneously in two mutually perpendicular oscillations  $x = 2 \sin \omega t$  m and  $y = 2 \cos \omega t$  m. Find the trajectory of the point.

6.10. A point participates in two oscillations of the same period with the same initial phases. Oscillation amplitudes:  $A_1 = 3$  cm and  $A_2 = 4$  cm. Find the amplitude of the resulting oscillation if: 1) the oscillations occur in one direction; 2) oscillations are mutually perpendicular. Represent these oscillatory movements graphically.

6.11. The point participates simultaneously in two mutually perpendicular oscillations:  $x = 2 \sin \omega t$  m and  $y = 2 \cos \omega t$  m. Find the trajectory of the oscillating point.

6.12. A material point simultaneously participates in two mutually perpendicular oscillations:  $x = \sin \pi t$  m and  $y = 2 \sin(\pi t + \frac{\pi}{2})$  m. Find the trajectory of the point and draw it on a convenient scale.

6.13. The total energy of a body performing harmonic oscillatory motion is equal to  $3 \cdot 10^{-5}$  J, the maximum force acting on the body is equal to  $1,5 \cdot 10^{-3}$  N. Write the equation of motion of the body if the oscillation period is 2 s and the initial phase is  $60^\circ$ .

6.14. The period of decaying oscillations is 4 s, the logarithmic decay decrement is 1,6 and the initial phase is zero. The displacement of the point from the equilibrium position at  $t = \frac{T}{4}$  is equal to 4,5 cm. Write the equation of motion of this oscillation.

6.15. A 24,7 cm long mathematical pendulum performs decaying oscillations. After how long will the energy of the pendulum's oscillations decrease by 9,4 times? Solve the problem at the value of the logarithmic decay decrement: 1) = 0,01; 2) = 1. Represent these oscillatory movements graphically.

6.16. A mathematical pendulum 0,5 m long, removed from the equilibrium position, deviated at the first oscillation by 5 cm, and at the second (in the same direction) - by 4 cm. Find the relaxation time, that is, the time during which the amplitude of oscillations will decrease by  $e$  times, where  $e$  is the base of the natural logarithm.

6.17. Determine the wavelength, frequency and period of natural oscillations of the circuit, which consists of a coil with an inductance of  $L = 10$  mH and parallel-connected capacitors with capacities  $C_1 = 880$  pF,  $C_2 = 20$  pF, respectively. ( $c = 3 \cdot 10^8$  m/s,  $\pi = 3.14$ ).

6.18. Determine the wavelength, frequency and period of natural oscillations of the circuit, which consists of a coil with an inductance of  $L = 40$  mH and parallel-connected capacitors with capacities  $C_1 = 600$  pF,  $C_2 = 400$  pF, respectively. ( $c = 3 \cdot 10^8$  m/s,  $\pi = 3.14$ ).

6.19. A rectangular frame rotates in a horizontal uniform field at a speed of  $n$  revolutions per second. Frame area  $S$ , magnetic induction  $B_0$ . Determine the law of change of the magnetic flux through the frame as a function of time, if the frame is placed horizontally at the initial moment.

6.20. The amplitude of the capacitor charge change in the oscillating circuit is equal to  $10 \mu\text{C}$ . What is the form of the equation for the change in current strength in the circuit, if the capacitor capacity is  $10 \mu\text{F}$ , and the coil inductance is  $0,1$  H?

6.21. The wave propagates at a speed of  $6$  m/s with a frequency of  $4$  Hz. What is the phase difference of wave points that are  $0,5$  m apart from each other?

6.22. Waves propagate along a rubber cord at a speed of  $3$  m/s at a frequency of  $2$  Hz. In what phases are the points that are  $75$  cm apart from each other?

6.23. The generator, operating at a frequency of  $50$  kHz, sends ultrasonic pulses lasting  $1/500$  s. How many ultrasonic waves are contained in one pulse?

6.24. Sound vibrations propagate in water at a speed of  $1480$  m/s, and in air at a speed of  $340$  m/s. How many times will the length of the sound wave change when sound passes from air to water?

6.25. The sound waves of the human voice have a length from  $33$  cm to  $4,0$  m. Determine the frequency of oscillations that corresponds to them.

6.26. The speed of sound in water is  $1450$  m/s. At what distance are the points oscillating in opposite phases when the frequency of oscillation is  $731$  Hz?

6.27. Determine the difference in the phases of oscillations of two points distant from the source of oscillations by  $3,5$  m and  $2$  m, respectively, if the oscillation period is  $0,5$  s and the oscillations propagate at a speed of  $6$  m/s.

6.28. At what frequency is the radio set, if its receiving circuit has an inductance of 1,5 mH and a capacity of 450 pF?

6.29. After what time will the signal reflected from the target return to the radar if the target is 50 km from the radar?

6.30. The frequency of electromagnetic oscillations generated by the radio transmitter is 6 MHz. What is the length of electromagnetic waves emitted by the radio?

6.31. At what frequency does the ship transmit the SOS distress signal if, by international agreement, the length of the radio wave should be 600 m?

6.32. The inductance of the oscillating circuit is 20  $\mu$ H. It is necessary to adjust this circuit to a frequency of 5 MHz. What capacity should be chosen?

6.33. What is the distance to the aircraft if the signal sent by the ground radar after it deflected from the aircraft returned to the radar in  $2 \cdot 10^{-4}$  s?

6.34. The radio signal sent to and reflected from the Moon was received on Earth 2,5 seconds after amplification. The same signal sent to Venus was received after 2,5 min. To determine the distance from Earth to the Moon and from Earth to Venus during the location.

6.35. The radar sends 2000 pulses per second. Determine the range of this locator.

6.36. The equation of decaying oscillations has the form  $x = 5 \cdot e^{-0,25t} \sin \frac{\pi}{2} t$  m. Determine the speed of the oscillating point at the instants of time: 0, T, 2T, 3T and 4T.

6.37. Logarithmic decrement of decay of mathematical pendulum is equal to 0,2. Find how many times the amplitude of oscillations will decrease in one complete oscillation of the pendulum.

6.38. What is the logarithmic decrement of the decay of a mathematical pendulum, if the amplitude of oscillations has decreased by two times in 1 min? The length of the pendulum is 1 m.

6.39. The amplitude of the attenuated oscillations of the mathematical pendulum was halved in 1 min. How many times will it decrease in 3 minutes.

6.40. In a circuit with an inductance of 40 mH and a capacity of 9  $\mu$ F, free undamped oscillations occur. Knowing that the maximum voltage on the capacitor is 100 V, determine the maximum current in this circuit.

6.41. The amplitude of the capacitor charge change in the oscillating circuit is equal to 10  $\mu$ C. What is the form of the equation for the change in current strength in the circuit, if the capacitor capacity is 10  $\mu$ F and the coil inductance is 0,1 H?

6.42. A resistor with a resistance of 20 Ohms and a capacitor with a reactance of 15 Ohms are connected in series in an alternating current circuit with a voltage of 250 V. Determine how many times the voltage drop across the capacitor is greater than the voltage drop across the resistor.

6.43. A resistor with a resistance of 4 ohms, a coil with an inductance of 2 mH and a capacitor with a capacity of 8  $\mu\text{F}$  are connected in series to an alternating current network with a voltage of 36 V and a frequency of 1 kHz. Find the voltage across the coil terminals to two significant figures ( $\pi = 3.14$ ).

6.44. Determine the cyclic frequency of oscillations in an electric oscillating circuit if the maximum current in the inductor is 1 A, the maximum potential difference on the capacitor plates is 300 V, and the energy of the circuit is 0.15 mJ.

6.45. Under the condition of the previous problem, determine the period and frequency of oscillations in the circuit.

6.46. After 0,25  $\mu\text{s}$  after switching on the oscillating circuit, the energy of the magnetic field of the coil became equal to the energy of the electric field of the capacitor. Determine the frequency of oscillations occurring in the circuit, if the current in the inductor coil changes according to the law  $I = I_0 \cdot \sin \omega t$ .

6.47. The oscillating circuit consists of a coil with an inductance of 2,5 mH and an air capacitor with a capacity of 10 pF. How many times will the frequency and period of oscillation change if the space between the capacitor plates is filled with bakelite?

6.48. Ultrasound is used to measure flow rates for liquid and gas. What is the speed of the flow, when the distance between the two vibrators is 100 m, ultrasound travels in one direction in 1 s and in the opposite direction - in 1,05 s?

6.49. Determine the distance between the nodes of a standing wave formed by a tuning fork in the air. Oscillation frequency is 425 Hz. The speed of sound is 340 m/s.

6.50. The current change in the oscillating circuit corresponds to the equation  $I = 0,3 \sin 15,7t$ . Determine the length of the electromagnetic wave emitted by the circuit.

6.51. The loudspeaker produces a sound wave with a length of  $\lambda = 3,4 \cdot 10^{-2}$  m. What is the length  $\lambda_0$  of the electromagnetic wave emitting an alternating current in the wires connected to the loudspeaker? The speed of sound is 340 m/s.

6.52. A tuning fork (440 Hz) sounds near the ear. At what distance from it should a second identical tuning fork be placed so that due to the interference of the waves, the ear does not feel the sound?

6.53. A steamboat moving along the river emits a sound signal with a frequency of  $\nu_0 = 400$  Hz. An observer standing on the shore perceives the sound of the whistle as an oscillation with a frequency of  $\nu = 395$  Hz. How fast is the ship moving?

6.54. At the hole of the copper pipe with a length of 366 m, a sound was formed that reached the second end of the pipe with metal 1 s earlier than air. What is the speed of sound in copper? The air temperature is 0°C.

6.55. A stone fell into the mine. A person heard the sound with which a stone fell 6 s after the beginning of the fall. Determine the depth of the mine. The speed of sound is 332 m/s.

6.56. The wave formed by the boat, which passed from the shore at a distance of 200 m, reached the shore in 100 seconds. What is the wavelength formed by the boat if the frequency of wave impacts on the shore is 0,5 Hz?

6.57. Will the wavelength of electromagnetic oscillations change in the closed oscillatory circuit if its capacitance is increased by 4 times, and its inductance is reduced by 4 times?

6.58. The circuit of the radio receiver is set to a wavelength of 20 m. How to change the inductance of the coil of the oscillatory circuit of the receiver so that it is set to a wavelength of 10 m?

6.59. The circuit of the radio receiver is set to a wavelength of 50 m. How to change the capacitance of the capacitor of the oscillatory circuit of the receiver so that it is set to a wavelength of 25 m?

6.60. The amplitude of the capacitor charge change in the oscillatory circuit is  $10 \mu\text{C}$ . What is the equation for changing the current in the circuit, if the capacitance of the capacitor is  $10 \mu\text{F}$ , and the inductance of the coil is  $0,1 \text{ H}$ ?

6.61. Determine the length of the wave if it propagates at a speed of  $5,5 \text{ m/s}$ , and its period is  $3 \text{ s}$ .

6.62. The wave length reaches  $270 \text{ m}$  in the ocean, the period is  $13,5 \text{ s}$ . Determine the speed of propagation of such a wave.

6.63. As a result of the explosion, which was carried out by geologists, a wave propagates in the earth's crust at a speed of  $5 \text{ km/s}$ . The wave reflected from the deep layers of the Earth was recorded  $22 \text{ s}$  after the explosion. At what depth (in km) does the rock lie, the density of which differs from the density of the earth's crust?

6.64. Infrasounds with a frequency of  $8 \text{ Hz}$  are harmful for human health. Determine the wavelength of this infrasound in air, if the speed of sound in air is  $340 \text{ m/s}$ .

6.65. The ultrasonic echo sounder creates oscillations with a frequency of  $40 \text{ kHz}$ . What is the depth of the sea if the ultrasound pulses of the sonar return  $0,2 \text{ s}$  after exiting? The ultrasonic wave length in water is  $0,035 \text{ m}$ .

6.66. What is the depth of the sea if the ultrasonic sonar signal returned  $0,4 \text{ s}$  after exiting? The speed of propagation of ultrasonic waves is assumed to be  $1500 \text{ m/s}$ .

6.67. The generator, which operates at a frequency of  $60 \text{ kHz}$ , sends ultrasonic pulses with duration of  $1/600 \text{ s}$ . How many wavelengths are in one pulse?

6.68. A sound wave passes from air to water. How many times longer is the sound wave length in water than in air? The speed of sound in water is  $1,496 \text{ m/s}$ , in air it is  $340 \text{ m/s}$ .

6.69. A person, standing on the seashore, determined that the distance between two crests of waves that follow each other is  $12 \text{ m}$ . In addition, the person

calculated that 16 wavecrests passed by him in 75 seconds. Determine the speed of wave propagation.

6.70. A wave created by a boat that passed from the shore at a distance of 200 m rolled to the shore in 80 s. What is the wavelength of the wave propagating from the boat, if the frequency of the waves hitting the shore is 0,5 Hz?

6.71. The steamer stirred up a wave that reached the shore after 1 minute. The distance between two adjacent wave humps is 1,5 m, and the time interval between two successive impacts on the shore is 2 s. What is the shortest distance from the shore to the steamer?

6.72. Compare the energy of waves of sound and ultrasonic frequency, if the amplitudes of oscillations are the same, and the frequencies are equal to  $\nu_1=1$  kHz and  $\nu_2=1$  MHz, respectively. When solving the problem, take into account that the energy of the particles of the medium is proportional to the square of the frequency of oscillations.

6.73. At what distance from the sound source is the obstacle, if the echo is heard 20 s after the sound? The speed of sound in air is 340 m/s.

6.74. The depth of the sea is measured by an ultrasonic sounder, which generates oscillations with a frequency of 50 kHz. What is the length of an ultrasonic wave in water? The speed of ultrasound in water is 1400 m/s.

6.75. How many times will the length of a sound wave change when it passes from air to water? The speed of sound propagation in air is 340 m/s, in water is 1450 m/s.

## SECTION 7. WAVE OPTICS AND THE QUANTUM NATURE OF RADIATION

### Basic laws and formulas

Condition of maximum light amplification during interference	$\Delta = \pm k\lambda$ ( $k = 1, 2, 3, \dots$ )
Condition of maximum light attenuation during interference	$\Delta = \pm (2k + 1)\lambda / 2$ ( $k = 1, 2, 3, \dots$ )
Interference band width	$\Delta x = \frac{L}{d}\lambda$
Optical path difference in reflected light in a thin film	$\Delta = 2d\sqrt{n^2 - \sin^2 i} + \lambda / 2$ $\Delta = 2dn \cos r + \lambda / 2$
Radius of Newton's light rings in reflected light	$r_k = \sqrt{(2k - 1)R \frac{\lambda}{2}},$ ( $k = 1, 2, 3, \dots$ ),

Radius of dark rings in reflected light	$r_k = \sqrt{kR\lambda}$ , ( $k = 1, 2, 3, \dots$ )
Radius of Fresnel zones for a spherical wave surface	$r_k = \sqrt{\frac{Rb}{R+b}} k\lambda$
Radius of Fresnel zones for flat wave front	$r_k = \sqrt{kb\lambda}$
Maximum condition for diffraction on one slit	$a \sin \varphi = \pm (2k + 1) \lambda / 2$
Minimum condition for diffraction on one slit	$a \sin \varphi = \pm 2k \frac{\lambda}{2}$
Maximum condition for diffraction on a diffraction grating	$d \sin \varphi = \pm 2k \frac{\lambda}{2}$
Minimum condition for diffraction on a diffraction grating	$d \sin \varphi = \pm (2k + 1) \lambda / 2$
Resolution of a diffraction grating	$R = \lambda / \Delta\lambda = kN$
Woolf-Bragg formula	$2d \sin \theta = k\lambda$
Brewster's Law	$\operatorname{tg} i = n_{2,1}$
Malus' law	$I = I_0 \cos^2 \varphi$
Angle of rotation of polarization plane in solutions	$\varphi = \alpha C d$
Stefan-Boltzmann law	$R^* = \sigma T^4$ , $\sigma = 5,67 \cdot 10^{-8} \text{ BT}/(\text{m}^2 \cdot \text{K}^4)$
Wien's Law of Displacement	$\lambda_{\max} = \frac{b}{T}$
Einstein's formula for external photoeffect	$h\nu = A + \frac{mv^2}{2}$
Compton's formula	$\Delta\lambda = \frac{2h}{m_0c} \sin^2 \frac{\theta}{2}$ , або $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c} (1 - \cos \theta)$
Light pressure	$p = \frac{I}{c} (1 - \rho)$ , $I = \frac{N h \nu}{S}$

### Examples of problem solving

**Problem 1.** To eliminate the reflection of light from the surface of the lens, a thin film of a substance with a refractive index of 1,25 is applied to it, which

is lower than that of glass (enlightened optics). At what is the smallest film thickness, the reflection of light with a wavelength of  $0,72 \mu\text{m}$  will not be observed if the angle of incidence of rays is  $60^\circ$ ?

**Given:**  $n = 1,25$ ,  $\lambda = 0,72 \text{ MKM}$ ,  $i = 60^\circ$

**Find:**  $d_{\min} - ?$

Optical difference in the path of rays reflected from the lower and upper surfaces of the film:

$$\Delta = 2d\sqrt{n^2 - \sin^2 i} . \quad (1)$$

In the expression (1) it is taken into account that the reflection of rays on both surfaces comes from an optically denser medium and therefore the half-minute losses in both cases compensate each other. Interference minimum condition:

$$\Delta = \pm(2k+1)\frac{\lambda}{2} . \quad (2)$$

Substituting (1) in (2), we obtain:

$$2d\sqrt{n^2 - \sin^2 i} = (2k+1)\frac{\lambda}{2} . \quad (3)$$

From (3) we will find possible values of film thickness:

$$d = \frac{(2k+1)\lambda}{4\sqrt{n^2 - \sin^2 i}} . \quad (4)$$

The lowest film thickness is observed at  $k=1$ :

$$d_{\min} = \frac{\lambda}{4\sqrt{n^2 - \sin^2 i}} ,$$

$$d_{\min} = \frac{0,75 \cdot 10^{-6}}{4\sqrt{1,25^2 - \sin^2 60^\circ}} = 0,2 \cdot 10^{-6} \text{ m} .$$

**Problem2.** The constant of the diffraction grating is  $10 \mu\text{m}$ , its width is  $2 \text{ cm}$ . In the spectrum of what order can this grating separate the doublet  $\lambda_1=486,0 \text{ nm}$  and  $\lambda_2=486,1 \text{ nm}$ ?

**Given:**  $d = 10 \mu\text{m}$ ,  $l = 2 \text{ cm}$ ,  $\lambda = 0,72 \mu\text{m}$ ,  $\lambda_1=486,0 \text{ nm}$ ,  $\lambda_2=486,1 \text{ nm}$

**Find:**  $k - ?$

Diffraction grating resolution:

$$R = \frac{\lambda}{\Delta\lambda} = kN , \quad (1)$$

where  $\Delta\lambda$  - minimum difference in wavelengths of two spectral lines  $\lambda$  and  $\lambda + \Delta\lambda$ , which are separated by a grid;  $k$  - the order of the spectrum;  $N$  - the

number of grating slots. Because the lattice constant  $d$  is the distance between the centers of adjacent slits, then:

$$N = \frac{l}{d}, \quad (2)$$

where  $l$  - grid width. From formula (1) taking into account (2) we find:

$$\Delta\lambda = \frac{\lambda}{kN} = \frac{d\lambda}{kl}. \quad (3)$$

The doublet of spectral lines  $\lambda_1$  and  $\lambda_2$  will be separated if:

$$\Delta\lambda \leq \lambda_2 - \lambda_1. \quad (4)$$

Substituting (3) into (4), taking into account the fact that  $\lambda = \lambda_1$ , we get:

$$\frac{d\lambda_1}{kl} \leq \lambda_2 - \lambda_1. \quad (5)$$

It follows from expression (5) that the doublet  $\lambda_1$  and  $\lambda_2$  will be allowed in all spectra with the order:

$$k \geq \frac{d\lambda_1}{l(\lambda_2 - \lambda_1)}.$$

Performing calculations, we get:

$$k \geq \frac{10 \cdot 10^{-6} \cdot 486,0 \cdot 10^{-9}}{2 \cdot 10^{-2} \cdot (486,1 - 486,0) \cdot 10^{-9}} = 2,43$$

Given that  $k$  should be an integer, we consider  $k \geq 3$ .

**Problem3.** The intensity of natural light passing through the polarizer decreased by 2,3 times. How many times will it decrease if a second polarizer of the same type is placed behind the first so that the angle between their main planes is equal to  $60^\circ$ ?

**Given:**  $\frac{I_0}{I_1} = 2,3, \alpha = 60^\circ$

**Find:**  $\frac{I_0}{I_2} - ?$

Natural light can be represented as a superposition of two incoherent waves polarized in mutually perpendicular planes and having the same intensity. An ideal polarizer passes oscillations parallel to its main plane and completely delays oscillations that are perpendicular to this plane. At the exit from the first polarizer, plane-polarized light is obtained, the intensity of which, taking into account losses due to reflection and absorption of light by the polarizer, is equal to:

$$I_1 = \frac{1}{2} I_0 (1 - k), \quad (1)$$

where  $I_0$  - intensity of natural light;  $k$  - coefficient, which takes into account reflection and absorption losses.

After passing through the second polarizer, the intensity of light decreases both due to reflection and absorption of light by the polarizer, and due to the non-coincidence of the plane of polarization of light with the main plane of the polarizer. According to the law of Malus and taking into account the losses due to reflection and absorption of light, this intensity is equal to:

$$I_2 = I_1(1-k)\cos^2\alpha, \quad (2)$$

where  $\alpha$  - angle between the polarization plane of light parallel to the main plane of the first polarizer and the main plane of the second polarizer. Let's find how many times the light intensity has decreased:

$$\frac{I_0}{I_2} = \frac{I_0}{I_1(1-k)\cos^2\alpha}. \quad (3)$$

From expression (1), we find:

$$(1-k) = \frac{2I_1}{I_0}, \quad (4)$$

Substituting (4) into (3), we get:

$$\frac{I_0}{I_2} = \frac{1}{2\cos^2\alpha} \left( \frac{I_0}{I_1} \right)^2.$$

Let's calculate:

$$\frac{I_0}{I_2} = \frac{1}{2\cos^2 60^\circ} (2,3)^2 = 10,6.$$

**Problem 4.** The wavelength at which the maximum energy occurs in the radiation spectrum of an absolutely black body is  $\lambda_{\max} = 0,58 \mu\text{m}$ . Determine the energy luminosity  $R^*$  of the body surface.

**Given:**  $\lambda_{\max} = 0,58 \mu\text{m}$

**Find:**  $R^* - ?$

The energy luminosity  $R^*$  of an absolutely black body, in accordance with the Stefan-Boltzmann law, is proportional to the fourth power of the thermodynamic temperature and is expressed by the formula:

$$R^* = \sigma T^4, \quad (1)$$

where  $\sigma$  - is the Stefan-Boltzmann constant;  $T$  - thermodynamic temperature.

The temperature  $T$  can be calculated using Wien's displacement law:

$$\lambda_{\max} = \frac{b}{T}, \quad (2)$$

where  $b$  - constant Vина.

Using formulas (2) and (1), we get:

$$R^* = \sigma \left( \frac{b}{\lambda_{max}} \right)^4. \quad (3)$$

Let's calculate the expression:

$$R^* = 5,67 \cdot 10^{-8} \left( \frac{2,90 \cdot 10^{-3}}{5,8 \cdot 10^{-7}} \right)^4 = 3,54 \cdot 10^7 \text{ W/m}^2.$$

**Problem 7.** As a result of the Compton effect, a photon colliding with an electron was scattered at an angle  $\theta = 90^\circ$ . Energy of scattered photon is  $\varepsilon_2 = 0,4 \text{ MeV}$ . Determine the energy of the photon  $\varepsilon_1$  before scattering.

**Given:**  $\theta = 90^\circ$ ,  $\varepsilon_2 = 0,4 \text{ MeV}$

**Find:**  $\varepsilon_1 - ?$

To determine the energy of the primary photon, use Compton's formula:

$$\Delta\lambda = 2 \frac{h}{m_0 c} \sin^2 \frac{\theta}{2}. \quad (1)$$

where  $\Delta\lambda = \lambda_2 - \lambda_1$  is the change in the wavelength of the photon as a result of scattering by a free electron;  $h$  - Planck's constant;  $m_0$  - rest mass of an electron;  $c$  - speed of light in vacuum;  $\theta$  - scattering angle of the photon.

Let's transform the formula (1):

- 1) let's replace  $\Delta\lambda$  with  $\lambda_2 - \lambda_1$ ;
- 2) let's express the wavelengths  $\lambda_1$  and  $\lambda_2$  in terms of energies  $\varepsilon_1$  and  $\varepsilon_2$  corresponding photons, using the formula  $\varepsilon = \frac{hc}{\lambda}$ ;
- 3) multiply the numerator and denominator of the right side by  $c$ . Then

$$\frac{hc}{\varepsilon_2} - \frac{hc}{\varepsilon_1} = \frac{hc}{m_0 c^2} 2 \sin^2 \frac{\theta}{2}.$$

Let's reduce it to  $hc$  and express the required energy from this formula:

$$\varepsilon_1 = \frac{\varepsilon_2 m_0 c^2}{m_0 c^2 - \varepsilon_2 2 \sin^2 (\theta/2)} = \frac{\varepsilon_2 E_0}{E_0 - 2\varepsilon_2 \sin^2 (\theta/2)}, \quad (2)$$

where  $E_0 = m_0 c^2$  is the rest energy of the electron.

As for the electron  $E_0 = 0,511 \text{ MeV}$ , then

$$\varepsilon_1 = \frac{0,4 \cdot 0,511}{0,511 - 2 \cdot 0,4 \sin^2 (90^\circ/2)} = 1,85 \text{ MeV}.$$

### Problems for self-solving

7.1. How many wavelengths of monochromatic light with a frequency of oscillations  $\nu = 5 \cdot 10^{10}$  Hz will fit on a path with a length of  $l = 1,2$  mm: 1) in a vacuum; 2) in glass?

7.2. Determine the length  $l_1$  of the segment on which the same number of wavelengths are deposited in a vacuum as are deposited on the segment  $l_2 = 3$  mm in water.

7.3. The path of which length  $l_1$  the front of the wave of monochromatic light will pass in a vacuum in the same time during which it passes the path of length  $l_2 = 1$  m in water?

7.4. A glass plate with thickness of  $h = 1$  mm was placed in the path of a light wave that propagates in the air. How much will the optical path length of light change if the wave falls on the plate: 1) normal; 2) at an angle  $i = 30^\circ$ ?

7.5. In the path of monochromatic light with a wavelength of  $\lambda = 0,6 \mu\text{m}$  is a flat-parallel glass plate with a thickness of  $d = 0,1$  mm. At what angle should the plate be turned so that the optical path length changes by  $1/2$ ?

7.6. How many times in Young's experiment do you need to change the distance to the screen so that the 5th light strip of the new interference picture turns out to be at the same distance in zero as the 3rd in the previous picture?

7.7. The distance between the slits in Young's experiment  $d = 0,5$  mm, the wavelength of light  $\lambda = 550$  nm. What is the distance from the slits to the screen, if the distance between adjacent dark stripes on it  $l = 1$  mm?

7.8. Young's experiment is performed in a transparent liquid with a refractive index  $n$ . Derive the ratio for the distance of the dark and light bands from the zero band.

7.9. The distance  $d$  between two coherent light sources ( $\lambda = 0,5 \mu\text{m}$ ) is  $0,1$  mm. The distance  $b$  between the interference bands on the screen in the middle part of the interference pattern is  $1$  cm. Determine the distance  $L$  from the source to the screen.

7.10. In Jung's experiment, the distance  $d$  between the slits is  $0,8$  mm. At what distance  $l$  from the slits should the screen be placed so that the width  $b$  of the interference band becomes equal to  $2$  mm?

7.11. In an experiment with Fresnel mirrors, the distance between the imaginary images of the light source  $d = 0,5$  mm. The distance to the screen  $L = 5$  m. When illuminated with green light on the screen received interference strips placed at a distance  $l = 5$  mm from each other. Find the wavelength  $\lambda$  of green light.

7.12. Normal monochromatic light falls on a diaphragm with two narrow slits located at a distance  $d = 2,5$  mm. An interference pattern is formed on a screen that is a distance away from the diaphragm  $l = 100$  cm. Where and to what distance

will the interference fringes shift if one of the slits is covered with a glass plate  $h = 1 \mu\text{m}$  thick and refractive index  $n = 1,5$ ?

7.13. In Jung's experiment, light with a wavelength  $\lambda_1 = 60 \text{ nm}$ , is first used, and then  $\lambda_2$ . What is the wavelength  $\lambda_2$ , if the 7th light band in the first case coincides with the 10th dark band in the second?

7.14. The distance  $L$  from the slit to the screen in Jung's experiment is 1 m. Determine the distance between the slits, if 10 dark interference bands are placed on a segment of the screen with a length  $l = 1 \text{ cm}$ . Wavelength  $\lambda = 0,7 \mu\text{m}$ .

7.15. Determine the thickness of the oil film on the water surface if radiation with a wavelength of  $\lambda = 0,589 \mu\text{m}$  is significantly increased when observed at an angle of  $60^\circ$  to normal in reflected light.

7.16. The radius of curvature of a flat-convex lens  $R = 4 \text{ m}$ . What is the wavelength  $\lambda$  of incident light equal if the radius of the 5th light ring in reflected light is  $3,6 \text{ mm}$ ?

7.17. Determine the radius of the 4th dark Newton ring, if between the lens the radius of curvature  $R = 5 \text{ m}$  and the flat surface to which it is pressed is water. Light wavelength  $\lambda = 589 \text{ nm}$ .

7.18. To observe Newton's rings, a flat convex lens with a radius of curvature  $R = 160 \text{ cm}$  is used. Determine the radii of the 4th and 9th dark rings in reflected light if the system is illuminated by monochromatic light with wavelength  $\lambda = 625 \text{ nm}$ .

7.19. The beam of monochromatic light waves falls at an angle  $i = 30^\circ$  on the soap film ( $n = 1,3$ ), which is contained in the air ( $\lambda = 0,6 \mu\text{m}$ ). At which the smallest film thickness  $d$  the reflected light waves will be maximally attenuated by interference; maximally enhanced?

7.20. The diameter of Newton's second light ring when observed in reflected light ( $\lambda = 0,6 \mu\text{m}$ ) is  $1,2 \text{ mm}$ . Determine the radius of curvature of the plano-convex lens taken for the experiment.

7.21. Flat convex lens with radius of curvature  $R = 0,5 \text{ m}$  lies on glass plate convex side. The radius  $r_4$  of Newton's fourth dark ring in light is  $0,7 \text{ mm}$ . Determine the length of the light wave.

7.22. At an angle  $\alpha = 30^\circ$ , the 4th diffraction maximum for wavelength  $\lambda = 0,644 \mu\text{m}$  is observed. Determine the constant of the diffraction grating and its width, if it allows to divide  $\Delta\lambda = 0,322 \text{ nm}$ .

7.23. Normal monochromatic light falls on a diffraction grating containing 100 strokes per 1 mm. The visual tube of the spectrometer is given at the maximum of the third order. To bring the pipe to another maximum of the same order, it must be turned by an angle  $\Delta\varphi = 20^\circ$ . Determine the wavelength of light.

7.24. The diffraction grating is illuminated by monochromatic light, which falls normally. In the diffraction pattern, the second order maximum is

deflected by an angle  $\varphi_1 = 14^\circ$ . What angle  $\varphi_2$  is the maximum of the third order deviated by?

7.25. The diffraction grating contains 200 strokes per 1 mm. Normal monochromatic light falls on the grating ( $\lambda = 0,6 \mu\text{m}$ ). What is the highest order maximum given by this lattice?

7.26. Parallel X-ray beam falls on edge of rock salt crystal ( $\lambda = 147 \text{ pm}$ ). Determine the distance between the atomic planes of the crystal if the diffraction maximum of the 2nd order is observed when the radiation falls at an angle  $\theta = 31^\circ 30'$  to the crystal surface.

7.27. Normal monochromatic light ( $\lambda = 0,5 \mu\text{m}$ ) falls on a diffraction grid containing 400 strokes per 1 mm. Find the total number of diffraction maxima that this lattice gives.

7.28. A parallel beam of X-ray radiation falls on the edge of the crystal. At an angle  $\theta = 65^\circ$  to the face plane, a first order maximum is observed. The distance  $d$  between the atomic planes of the crystal 280 pm. Determine the wavelength of X-ray radiation.

7.29. Brewster's angle  $i$  when light falls from the air onto a rock salt crystal is  $57^\circ$ . Determine the speed of light  $v$  in this crystal.

7.30. The analyzer weakens the intensity of polarized light falling on it by 2 times. Determine the angle between the main planes of the polarizer and the analyzer. The losses of light on reflection are neglected.

7.31. The natural light beam passes sequentially through the polarizer and analyzer, the angle between the main planes of which is  $60^\circ$ . How much of the initial flow will come out of the analyzer?

7.32. The angle between the main planes of the polarizer and the analyzer is  $45^\circ$ . How many times will the intensity of the light coming out of the analyzer decrease if the angle is increased to  $60^\circ$ ?

7.33. How many times will natural light weaken, passing through two Nicol prisms, the main planes of which are located at an angle of  $63^\circ$ , if 10% of the incident light is lost in each of these prisms.

7.34. Determine the temperature  $T$ , at which the energy luminosity  $R_e$  of an absolutely black body is equal to  $10 \text{ kW/m}^2$ .

7.35. Determine the energy  $W$  emitted in a time  $t = 1 \text{ min}$ . from the inspection window with an area  $S = 8 \text{ cm}^2$  of the melting furnace, if its temperature is  $T = 1200 \text{ K}$ .

7.36. How many times do you need to increase the thermodynamic temperature of an absolutely black body so that its energy luminosity  $R_e$  increased by two times?

7.37. From the surface of soot  $S = 2 \text{ cm}^2$  at a temperature  $T = 400 \text{ K}$  for a time  $t = 5 \text{ min}$  energy is emitted  $W = 83 \text{ J}$ . Determine the blackness coefficient of soot.

7.38. Find the power emitted by an absolutely black ball with a radius  $r = 10$  cm. Find the power emitted by an absolutely black ball with a radius  $r = 20$  °C.

7.39. The temperature of an absolutely black body varies from 727 °C to 1727 °C. How many times will the energy emitted by the body change?

7.40. The temperature of an absolutely black body is 127 °C. After increasing the temperature, the total radiation power increased by 3 times. How much did the temperature increase?

7.41. How many times will the radiation power of an absolutely black body increase if the wavelength of the maximum radiation changes from  $\lambda_{1m} = 700$  nm to  $\lambda_{2m} = 600$  nm?

7.42. Neglecting heat conduction losses, find the power of the electric current, which is required to heat a thread with a diameter of 1 mm and a length of 20 cm to a temperature of 2500 K. Consider that the thread radiates as a completely black body and all the heat released in it goes to radiation after establishing equilibrium.

7.43. Determine the wavelength of radiation if its corresponding photons have energy  $\varepsilon = 10^{-19}$  J.

7.44. Determine the mass of the radiation photon with wavelength  $\lambda = 280$  nm.

7.45. Photon energy  $\varepsilon = 10$  J. Determine the momentum of the photon.

7.46. What wavelength must a photon have so that its mass is equal to the mass of an electron that is at rest?

7.47. Determine the energy and momentum of a photon whose wavelength  $\lambda = 500$  nm.

7.48. Will a photoeffect be observed if ultraviolet radiation with a wavelength  $\lambda = 300$  nm is directed at the surface of silver?

7.49. What fraction of the photon energy is spent on the operation of the photoelectron extraction if the red limit of the photoeffect is 307 nm and the maximum kinetic energy of the photoelectron is 1 eV.

7.50. Monochromatic light falls on the lithium surface ( $\lambda = 310$  nm). To stop the emission of electrons, it is necessary to apply a delaying potential difference of at least 1,7 V. Determine the work of the electron output.

7.51. Monochromatic light with a wavelength falls on a zinc plate  $\lambda = 220$  nm. Determine the maximum speed of photoelectrons.

7.52. What frequency of electromagnetic radiation should be directed at the surface of platinum so that the maximum speed of electrons is equal to 100 m/s?

7.53. Determine the red limit of the photoeffect for zinc and the maximum speed of photoelectrons ejected from its surface by electromagnetic radiation, the wavelength of which is  $\lambda = 250$  nm.

7.54. By alternating illumination of the surface of a certain metal with light with wavelengths  $\lambda_1 = 0,35 \mu\text{m}$  and  $\lambda_2 = 0,54 \mu\text{m}$ , it was found that the corresponding maximum velocities of electrons differ by 2 times from each other. Find the work of emitting electrons from the surface of this metal.

7.55. Find Planck's constant, if the electrons ejected from the metal by light with a frequency of  $\nu_1 = 2,2 \cdot 10^{15} \text{ Hz}$ , are completely delayed by a potential difference of  $U_1 = 6,6 \text{ V}$  and those ejected by light with a frequency of  $\nu_2 = 4,6 \cdot 10^{15} \text{ Hz}$  - by a potential difference  $U_2 = 16,5 \text{ V}$ .

7.56. The wavelength of  $332 \text{ nm}$  corresponds to the red limit of the photoelectric effect for aluminum. Find the work output of an electron from this metal and the wavelength of the radiation incident on the surface of the metal if the corresponding stopping voltage for photoelectrons is  $1 \text{ V}$ .

7.57. Electrons are completely detained by a potential difference of  $0,8 \text{ V}$  during the photoeffect from the platinum surface. Find the wavelength of the radiation  $\lambda$  and the limiting wavelength  $\lambda_0$  at which the photoeffect is still possible.

7.58. Determine the scattering angle of a photon  $\theta$  that has collided with a free electron (Compton effect), if the change in wavelength  $\Delta\lambda$  upon scattering is equal to  $3,62 \text{ pm}$ .

7.59. X-ray radiation of wavelengths  $\lambda = 55,8 \text{ pm}$  is scattered by graphite (Compton effect). Determine the wavelength of light  $\lambda'$  scattered at an angle  $\theta = 60^\circ$  to the direction of the incident beam.

7.60. The photon with energy  $\varepsilon = 0,4 \text{ MeV}$  dissipated at an angle  $\theta = 90^\circ$  on a free electron. Determine the energy of the scattered photon  $\varepsilon'$  and the kinetic energy of the recoil electron  $E_k$ .

7.61. X-ray radiation with wavelength  $\lambda = 56,3 \text{ pm}$  is scattered by paraffin tile. Determine the wavelength of rays scattered at an angle of  $120^\circ$  to the initial direction of X-ray radiation.

7.62. What was the X-ray wavelength if the Compton scattering of this radiation by graphite at an angle  $\varphi = 60^\circ$  the wavelength of the scattered radiation was equal to  $25,4 \text{ pm}$ ?

7.63. The photon with energy  $\varepsilon = 0,75 \text{ MeV}$  dissipated on a free electron at an angle  $\varphi = 45^\circ$ . Find the energy of the scattered photon  $\varepsilon'$ , the kinetic energy  $E_k$  and the momentum of the recoil electron  $p$ .

7.64. Radiation pressure on the flat mirror  $p = 0,2 \text{ Pa}$ . Determine the intensity of light incident on the surface of this mirror if its reflection coefficient is  $0,6$ . The luminous flux falls on the surface of the mirror normally.

7.65. A parallel beam of light with an intensity  $I = 0,2 \text{ W/cm}^2$  falls normally on a flat mirror with a reflection coefficient of 0,9. Determine the pressure of light on the mirror.

7.66. Determine the radiation pressure with wavelength  $\lambda = 0,5 \mu\text{m}$  on a blackened plate, if an energy  $W = 0,005 \text{ J}$  falls on a unit surface of the plate in  $t = 1 \text{ s}$ ? The reflectance of the plate is zero.

7.67. Determine the number of photons that fall over time  $t = 1$  second on the surface of paper with an area of  $1 \text{ cm}^2$  in the flow of monochromatic radiation ( $\lambda = 0,63 \mu\text{m}$ ), if the radiation pressure on the paper  $p = 2 \mu\text{Pa}$ . The reflection coefficient of the paper is 0,2.

7.68. The wavelength of radiation that corresponds to the red limit of the photoeffect for sodium is 530 nm. What is the work of the output of electrons from sodium (give the result in J)?

7.69. What is the momentum and wavelength of radiation whose photons have a mass of  $4,0 \cdot 10^{-36} \text{ kg}$ ?

7.70. The work of the output of electrons from tungsten is 4,5 eV. What is the minimum frequency of electromagnetic radiation capable of causing a photoeffect during irradiation of the tungsten surface?

7.71. X-ray photons have an energy of 50 keV. What is their wavelength and what is their mass?

7.72. What is the maximum wavelength of electromagnetic radiation capable of causing a photoeffect when irradiating a zinc plate? The output from zinc is 4,2 eV.

7.73. What are the energy and momentum of infrared photons with a frequency of 30 THz?

7.74. The work of the electron output from platinum is 5,3 eV. What is the minimum frequency of radiation, due to which platinum absorption a photoeffect is possible?

7.75. What is the wavelength of X-ray radiation, the frequency of which is  $1,0 \cdot 10^{17} \text{ Hz}$ ? What is the momentum of the photons in this case?

## SECTION 8. ATOMIC AND NUCLEAR PHYSICS

### Basic laws and formulas

General formula of radiation series of hydrogen-like systems	$\frac{1}{\lambda} = RZ^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ $n_2 = n_1 + 1; n_1 + 2; n_1 + 3; \dots$
Photon energy	$\varepsilon = \frac{hc}{\lambda} = R' Z^2 hc \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ $R = R' c = 1,097 \cdot 10^7 c$
Ionization energy	$\varepsilon_i = R' hc = 13,6 \cdot Z^2 eV$
Total energy of the electron	$E = E_K + E_{II} = -\frac{1}{n^2} \frac{Z^2 m e^4}{8 h^2 \varepsilon_0^2}$ $n = 1, 2, 3, \dots$
Orbital radius	$r_n = n^2 \frac{4 \pi \varepsilon_0 \hbar^2}{m e^2}$
De Broglie wavelength	$\lambda = \frac{h}{p} = \frac{h}{mv}$
Momentum is non-relativistic	$p = \sqrt{2 m_0 E_K}; E_K = \frac{m v^2}{2} = \frac{p^2}{2m}$
Momentum relativistic	$p = \frac{1}{c} \sqrt{E_K (E_K + 2 E_0)};$ $E_0 = m_0 c^2$
Heisenberg uncertainty ratio	$\Delta x \cdot \Delta p \geq \hbar = \frac{h}{2\pi}, \quad \Delta E \cdot \Delta t \geq \hbar$
Density of probability of detection of a particle in a potential well	$\omega =  \psi ^2 = \left( \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l} x \right)^2$
Wave function describing the state of the microparticle in an infinitely deep one-dimensional potential well width $l$	$\psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi \cdot x}{l}$
Total microparticle energy in potential well width $l$	$E = n^2 \frac{h^2}{8 m l^2}; n = 1, 2, 3, \dots$

Quantization of orbital moment of electron pulse	$L_l = \hbar \sqrt{l(l+1)}$
Quantization of the spin moment of the pulse (spin) of the electron	$L_s = \hbar \sqrt{s(s+1)}$
Orbital magnetic moment of the electron	$\mu_l = \mu_B \sqrt{l(l+1)}$
Spin magnetic moment of the electron	$\mu_s = 2\mu_B \sqrt{s(s+1)}$
Short-wavelength limit of X-ray radiation	$\lambda_{min} = \frac{hc}{eU}$
Moseley's Law	$\frac{1}{\lambda} = R(Z - \sigma)^2 \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$
Law of radioactive decay	$N = N_0 e^{-\lambda t}; N_0 = \frac{m}{M} N_A$
Half-life	$T = \frac{\ln 2}{\lambda}$
Average life time of a radioactive nuclide	$\tau = \frac{1}{\lambda}$
Activity of radioactive substance	$a = a_0 e^{-\lambda T}; a_0 = \lambda N_0$
Law of absorption of gamma radiation by substance	$I = I_0 e^{-\mu \cdot x}$
Atomic core mass defect	$\Delta m = Zm_p + (A - Z)m_n - m_x$
Core bond energy	$E_{36} = c^2 \Delta m$
Nuclear reaction energy	$Q = c^2 (\Sigma M_1 - \Sigma M_2)$

### Examples of problem solving

**Problem 1.** Camerton oscillates with frequency  $\nu_0 = 800$  Hz and amplitude  $A = 4$  mm. Find the maximum acceleration of its oscillating branch.

**Given:**  $\nu_0 = 800$  Hz,  $A = 4$  mm.

**Find:**  $a_{max} = ?$

The tuning fork branch motion equation has the form (in SI system):

$$x = A \sin(2\pi\nu_0 t + \varphi) = 0,004 \sin(2\pi 800 t + \varphi). \quad (1)$$

**Problem 2.** The electron in the hydrogen atom went from the fourth energy level to the second. Determine the energy of the photon released in this case.

**Given:**  $n_1 = 2, n_2 = 4$

**Find:**  $\varepsilon - ?$

To determine the energy of a photon, use the serial formula for water-like systems:

$$\frac{1}{\lambda} = RZ^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad (1)$$

where  $\lambda$  - photon wavelength;  $R$  - Rydberg's constant;  $Z = 1$  - core charge;  $n_1 = 2$  - number of the orbit to which the electron moved;  $n_2 = 4$  - number of the orbit from which the electron moved.

The energy of a photon is given by the formula:

$$\varepsilon = \frac{hc}{\lambda} = R' Z^2 hc \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right). \quad (2)$$

Since the value  $\varepsilon_i = R' hc = 13,6 \text{ eV}$  is the ionization energy of a hydrogen atom, then:

$$\varepsilon = \varepsilon_i Z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right). \quad (3)$$

After making calculations using formula (3), we get:

$$\varepsilon = 13,6 \cdot 1^2 \left( \frac{1}{2^2} - \frac{1}{4^2} \right) = 2,55 \text{ eV}.$$

**Problem 3.** Find angular velocity  $\omega$  and rotation period  $T$  of the electron in the first orbit of Bohr in the hydrogen atom.

**Given:**  $m = 9,1 \cdot 10^{-31} \text{ kg}, n = 1$

**Find:**  $\omega, T - ?$

According to Bohr's first postulate:

$$mv \cdot r = n \frac{h}{2\pi}, \quad (1)$$

where  $m = 9,1 \cdot 10^{-31} \text{ kg}$  - electron mass,  $r$  - orbit radius,  $v$  - linear velocity of the electron in this orbit,  $h$  - Planck's constant,  $n = 1$  - a quantum number that corresponds to the first orbit.

Considering that  $v = \omega \cdot r$ , we can write:

$$m\omega \cdot r^2 = n \frac{h}{2\pi} . \quad (2)$$

The radius of the orbit is defined by the formula:

$$r_n = n^2 \frac{\varepsilon_0 h^2}{\pi m e^2} , \quad (3)$$

where  $e$  - electron charge,  $\varepsilon_0$  - electrical constant.

Substituting formula (3) into (2), we get:

$$\omega = \frac{\pi m e^4}{2 \varepsilon_0^2 n^3 h^3} . \quad (4)$$

Substituting numerical values into formula (4), we get:

$$\omega = \frac{3,14 \cdot 9,1 \cdot 10^{-31} \cdot (1,6 \cdot 10^{-19})^4}{2 \cdot (8,85 \cdot 10^{-12})^2 \cdot (6,62 \cdot 10^{-34})^3} = 4,4 \cdot 10^{16} \text{ rad/sec.}$$

The rotation period of the electron is found by the equation:

$$T = \frac{2\pi}{\omega} . \quad (5)$$

After performing calculations on equation (5), we will find:

$$T = \frac{6,28}{4,4 \cdot 10^{16}} = 1,43 \cdot 10^{-16} \text{ s.}$$

**Problem 4.** The electron, whose initial velocity can be neglected, passed by accelerating the difference in potentials  $U$ . Find de Broglie wavelength for two cases: 1)  $U_1 = 51 \text{ V}$ ; 2)  $U_2 = 510 \text{ kV}$ .

**Given:**  $m = 9,1 \cdot 10^{-31} \text{ kg}$ ,  $U_1 = 51 \text{ V}$ ;  $U_2 = 510 \text{ kV}$

**Find:**  $\lambda_1, \lambda_2 - ?$

The de Broglie wavelength for a particle depends on its momentum  $p$  and is defined by the formula:

$$\lambda = \frac{h}{p} , \quad (1)$$

where  $h$  - Planck's constant.

The momentum of a particle can be determined if its kinetic energy is known  $E_K$ . The association of momentum with kinetic energy is not the same for the non-relativistic case (when the kinetic energy of a particle is much less than its rest energy) and for the relativistic case (when the kinetic energy is combined with the rest energy of the particle).

In the non-relativistic case:

$$p = \sqrt{2m_0 E_K} ; \quad (2)$$

where  $m_0 = 9,1 \cdot 10^{-31}$  kg - rest mass of a particle.

In the relativistic case:

$$p = \frac{1}{c} \sqrt{E_K (E_K + 2E_0)}, \quad (3)$$

where  $E_0 = m_0 c^2$  - rest energy of a particle.

The formula (1) taking into account the relations (2) and (3) in the non-relativistic case is written:

$$\lambda = \frac{h}{\sqrt{2m_0 E_K}}, \quad (4)$$

in the relativistic case:

$$\lambda = \frac{h}{\frac{1}{c} \sqrt{(2E_0 + E_K) E_K}}. \quad (5)$$

Compare the kinetic energies of an electron that has passed the potential difference  $U_1 = 51$  V and  $U_2 = 510$  kV with the rest energy of the electron and, depending on this, clarify which of the formulae (4) and (5) should be used to calculate the wavelength of de Broglie.

As you know, the kinetic energy of an electron that has undergone an accelerating potential difference  $U$ :

$$E_K = eU.$$

In the first case  $E_{K1} = eU_1 = 51 \text{ eV} = 0,51 \cdot 10^{-4} \text{ MeV}$ , which is much less than the rest energy of the electron  $E_0 = m_0 c^2 = 0,51 \text{ MeV}$ . Therefore, in this case, use formula (4). To simplify calculations, note that  $E_{K1} = 10^{-4} m_0 c^2$ . Substituting this expression into formula (4), rewrite it as:

$$\lambda_1 = \frac{h}{\sqrt{2m_0 \cdot 10^{-4} m_0 c^2}} = \frac{h}{\sqrt{2m_0^2 c^2 \cdot 10^{-4}}}. \quad (6)$$

Calculate by formula (6):

$$\lambda_1 = \frac{6,62 \cdot 10^{-34}}{\sqrt{2 \cdot (9,1 \cdot 10^{-31})^2 \cdot (3 \cdot 10^8)^2 \cdot 10^{-4}}} = 171 \text{ pm}.$$

In the second case, the kinetic energy is:

$$E_{K2} = eU_2 = 510 \text{ keV} = 0,51 \text{ MeV},$$

that is, equal to the rest energy of the electron. In this case, it is necessary to apply the relativistic formula (5).

Considering that  $E_{K2} = 0,51 \text{ MeV} = m_0 c^2$ , by formula (5) we find:

$$\lambda_2 = \frac{h}{\frac{1}{c} \sqrt{(2m_0c^2 + m_0c^2)m_0c^2}} = \frac{h}{\sqrt{3}m_0c^2}. \quad (7)$$

Calculate by formula (7):

$$\lambda_2 = \frac{6,62 \cdot 10^{-34}}{\sqrt{3} \cdot 9,1 \cdot 10^{-31} \cdot (3 \cdot 10^8)^2} = 1,4 \text{ pm}.$$

**Problem 5.** The kinetic energy of the electron in the hydrogen atom  $E_K$  is 10 eV. Using the uncertainty ratio, estimate the minimum linear dimensions of the atom.

**Given:**  $E_K = 10 \text{ eV}$ ,  $m_0 = 9,1 \cdot 10^{-31} \text{ kg}$

**Find:**  $l_{\min} - ?$

The uncertainty ratio for the coordinate and momentum is:

$$\Delta x \cdot \Delta p \geq \hbar = \frac{h}{2\pi}, \quad (1)$$

where  $\Delta x$  - uncertainty of the particle coordinate (in this case the electron);  $\Delta p$  - particle momentum uncertainty (electron);  $h$  - Planck's constant.

From the uncertainty ratio, it follows that the more precisely the position of the particle in space is determined, the more uncertain the momentum becomes, and, accordingly, the energy of the particle. Let the atom have linear dimensions  $l$ , then the electron of the atom will be within the region with uncertainty:

$$\Delta x = \frac{l}{2}. \quad (2)$$

The uncertainty ratio (2) can be written in this case as

$$\frac{l}{2} \Delta p \geq \hbar.$$

Where from:

$$l \geq \frac{2\hbar}{\Delta p}. \quad (3)$$

The uncertainty of the pulse  $\Delta p$ , in any case, should not exceed the value of the pulse itself, that is:

$$\Delta p \leq p.$$

Momentum  $p$  is related to kinetic energy  $E_K$  by the ratio:

$$p = \sqrt{2m_0E_K}.$$

Let's replace  $\Delta p$  with a value  $\sqrt{2m_0E_K}$ . Moving from inequality to equality, we get:

$$l_{min} = \frac{2\hbar}{\sqrt{2m_0 E_K}} . \quad (4)$$

Substitute numerical values into equation (4) and perform the calculation:

$$l_{min} = \frac{2 \cdot 1,05 \cdot 10^{-34}}{\sqrt{2 \cdot 9,1 \cdot 10^{-31} \cdot 1,6 \cdot 10^{-19} \cdot 10}} = 1,24 \cdot 10^{-10} \text{ m} = 124 \text{ pm}.$$

**Problem 6.** Determine the possible values of the orbital moment of the electron pulse  $L_l$  in the excited hydrogen atom if the excitation energy  $\varepsilon = 12,09 \text{ eV}$ .

**Given:**  $\varepsilon = 12,09 \text{ eV}$

**Find:**  $L_l - ?$

Orbital moment of the electron pulse  $L_l$  is determined by a quantum number  $l$  by the formula:

$$L_l = \hbar \sqrt{l(l+1)}, \quad (1)$$

where  $l$  - orbital quantum number ( $l=0,1,2,\dots, n-1$ );  $\hbar = h / 2\pi = 1,06 \cdot 10^{-34} \text{ J}\cdot\text{s}$ .

Since a number of possible values  $l$  is limited by the value  $n-1$ , we find the main quantum number  $n$  by the formula:

$$E_n = -me^4 / 32\pi^2 \varepsilon_0^2 \hbar^2 n^2 . \quad (2)$$

The formula (2) will be rewritten, taking into account that at  $n = 1$   $E_1 = -13,6 \text{ eV}$ :

$$E_n = -\frac{13,6}{n^2} \text{ eV}. \quad (3)$$

Excitation energy  $\varepsilon$  there is a quantum of energy absorbed by the atom in the transition from the ground state ( $n = 1$ ) to the excited one. Therefore:

$$E_n - E_1 = \varepsilon . \quad (4)$$

By substituting the numerical values of the quantities expressed in electron volts, we get:

$$-\frac{13,6}{n^2} + 13,6 = 12,09 ,$$

from where  $n = 3$ . Accordingly  $l = 0, 1, 2$ .

By formula (1) we find possible values  $L_l$ :

at  $l = 0$   $L_l = 0$ ;

at  $l = 1$   $L_l = \hbar \sqrt{2} = 1,49 \cdot 10^{-34} \text{ J}\cdot\text{s}$ ;

at  $l = 2$   $L_l = \hbar \sqrt{6} = 2,6 \cdot 10^{-34} \text{ J}\cdot\text{s}$ .

**Problem 7.** Calculate the mass defect and binding energy of the nucleus  ${}^7_3\text{Li}$ .

**Given:**  ${}^7_3\text{Li}$

**Find:**  $\Delta m, E_{36} - ?$

The mass of the nucleus is always less than the sum of the masses of free protons and neutrons (located outside the nucleus) from which the nucleus was formed.

The defect in the mass of the nucleus ( $\Delta m$ ) and is the difference between the sum of the masses of free nucleons (protons and neutrons) and the mass of the nucleus, that is:

$$\Delta m = Zm_p + (A - Z)m_n - m_a, \quad (1)$$

where  $Z$  - atomic number (number of protons in the nucleus);  $A$  - mass number (number of nucleons in the nucleus);  $m_p, m_n, m_a$  - respectively, the masses of the proton, neutron, and nucleus.

Reference tables always give masses of neutral atoms, but not nuclei. Therefore, it is advisable to change formula (1) so that it includes the mass  $M$  of a neutral atom.

It can be considered that the mass of a neutral atom is equal to the sum of the masses of the nucleus and electron that make up the electron shell of the

$$\text{atom: } M = m_a + Zm_e,$$

from where:

$$m_a = M - Zm_e. \quad (2)$$

Substituting formula (2) into (1), we get:

$$\Delta m = Zm_p + (A - Z)m_n - M + Zm_e,$$

or:

$$\Delta m = Z(m_p + m_e) + (A - Z)m_n - M.$$

Noticing that  $m_p + m_e = M_H$ , where  $M_H$  - the mass of the hydrogen atom, at the end we will find:

$$\Delta m = ZM_H + (A - Z)m_n - M. \quad (3)$$

Substituting numerical values of masses (according to reference data) into the expression (3), we get:

$$\Delta m = [3 \cdot 1,00783 + (7 - 3) \cdot 1,00867 - 7,01601] = 0,04216 \text{ a.m.u.}$$

The binding energy of a nucleus  $E_{36}$  is called energy, which in one form or another is released when the nucleus is formed from free nucleons.

According to the law of proportionality of mass and energy:

$$E_{36} = c^2 \Delta m, \quad (4)$$

where  $c$  - speed of light in vacuum.

The proportionality coefficient  $c^2$  can be expressed twofold:

$$c^2 = 9 \cdot 10^{16} \text{ m}^2/\text{s}^2,$$

or:

$$c^2 = \frac{\Delta E_{36}}{\Delta m} = 9 \cdot 10^{16} \text{ J/kg}.$$

If you calculate the energy of the connection using extrasystem units, then  $c^2 = 931 \text{ MeV/a.m.u.}$

Taking this into account, formula (4) will take on the form:

$$E_{36} = 931 \Delta m \text{ MeV.} \quad (5)$$

By substituting the previously found value of the core mass defect into formula (5), we get:

$$E_{36} = 931 \cdot 0,4216 = 39,2 \text{ MeV}.$$

### Problems for self-solving

8.1. Determine the radii  $r_2$  and  $r_3$  of the second and third orbits in the hydrogen atom.

8.2. Determine the speed of the electron in the second orbit of the hydrogen atom.

8.3. Determine the frequency of rotation of the electron in the second orbit of the hydrogen atom.

8.4. Determine the wavelength that corresponds to the third spectral line in the Balmer series.

8.5. Determine the energy of a photon released during the transition of an electron in a hydrogen atom from the third energy level to the first.

8.6. Find the largest wavelength in the first infrared series of the hydrogen spectrum (Paschen series).

8.7. Find the smallest wavelength in the first infrared series of the hydrogen spectrum (Paschen series).

8.8. Determine the lowest photon energy in the ultraviolet series of the hydrogen spectrum (Lyman series).

8.9. Determine the highest photon energy in the ultraviolet series of the hydrogen spectrum (Lyman series).

8.10. Using Bohr's theory, determine the radius of the first orbit in a hydrogen atom.

8.11. Using Bohr's theory, determine the speed and acceleration of an electron in the first orbit of a hydrogen atom.

8.12. Using Bohr's theory, determine the ratio of the radii of the orbits for the electron in the first and second orbits in the hydrogen atom.

8.13. Determine the frequency of radiation during the transition of an electron in a hydrogen atom from the third orbit to the second.

- 8.14. Determine the energy of a photon released during the transition of an electron in a hydrogen atom from the third energy level to the second.
- 8.15. Determine the maximum energy of a photon in the visible series of the hydrogen spectrum (Balmer series).
- 8.16. Determine the wavelength that corresponds to the second spectral line in Paschen's series.
- 8.17. Determine the wavelength of the spectral line that corresponds to the transition of an electron in a hydrogen atom from the sixth Bohr orbit to the second.
- 8.18. Determine the speed of an electron in the third orbit of a hydrogen atom.
- 8.19. Determine the rotation frequency of the electron in the third orbit of the hydrogen atom.
- 8.20. Determine the rotation frequency of the electron in the first orbit of the hydrogen atom.
- 8.21. Determine the minimum energy of a photon in the visible series of the hydrogen spectrum (Balmer series).
- 8.22. Find the de Broglie wavelength for electrons that have passed through a potential difference of 1 V; 100 V; 1000 V.
- 8.23. Find the de Broglie wavelength for electrons that travel at 108 m/s.
- 8.24. When the energy of the electron increased by 200 eV, its de Broglie wavelength changed twice. Find the initial wavelength of the electron.
- 8.25. Find the ratio of de Broglie waves for an electron and a proton accelerated by the same potential difference.
- 8.26. The initial de Broglie wavelength of the electron is 20 pm. What kind of energy should be given to the electron so that this wavelength doubles?
- 8.27. Determine the de Broglie wavelength for an electron that has passed an accelerating potential difference of 400 V.
- 8.28. Find the de Broglie wavelength for a hydrogen atom moving with a speed equal to the average quadratic speed at 300 K.
- 8.29. Find the de Broglie wavelength for a proton that has passed the accelerating potential difference of 1 MV.
- 8.30. The electron, whose initial velocity can be neglected, passed the accelerating potential difference of 51 V. Find the de Broglie wavelength.
- 8.31. Find the de Broglie wavelength for a proton that has passed through an accelerating potential difference of 1 kV.
- 8.32. The electron, whose initial velocity can be neglected, passed through an accelerating potential difference of 510 kV. Find the de Broglie wavelength.
- 8.33. What accelerating potential difference must an electron go through so that the de Broglie wavelength is equal to 0,1 nm?
- 8.34. Determine the de Broglie wavelength for a free electron moving at a speed of 106 m/s.

8.35. What is the acceleration potential difference that an electron must pass in order for de Broglie's wavelength to be 0,1 nm?

8.36. The kinetic energy of the electron is 0,6 MeV. Determine de Broglie wavelength.

8.37. Determine at what numerical speed value the de Broglie wavelength for an electron is equal to its Compton wavelength.

8.38. Determine the de Broglie wavelength for an electron if its kinetic energy is 1 keV; 1 MeV.

8.39. Based on the Heisenberg uncertainty ratio; estimate the size of the nucleus of the atom, believing that the minimum energy of the nucleon in the nucleus is 8 MeB.

8.40. Using the Heisenberg uncertainty ratio, estimate the energy of an electron in the first Bohr orbit in the hydrogen atom.

8.41. Estimate the uncertainty of the speed of the electron in the hydrogen atom, assuming the size of the atom of order  $10^{-8}$  cm.

8.42. What is the uncertainty of the momentum of the electron in the atomic nucleus ( $R = 10^{-15}$  cm)?

8.43. Assuming that the electron is inside an atom with a diameter of 0.3 nm, determine (in electron volts) the uncertainty of the energy of this electron.

8.44. The kinetic energy of the electron in the hydrogen atom is 10 eV. Using the uncertainty ratio, estimate the minimum linear dimensions of the atom.

8.45. Determine the inaccuracy in the determined coordinates of an electron moving in a hydrogen atom at a speed of  $1,5 \cdot 10^6$  m/s, if the permissible inaccuracy in the determined velocities is 10% of its magnitude.

8.46. The typical existence time of an atom in an excited state is 10 ns. Determine the uncertainty of the energy of the photon emitted during the transition of the atom to the normal state.

8.47. The electron is in an infinitely deep one-dimensional potential well with a width of 0,15 nm. Determine the lowest energy of an electron.

8.48. Determine how many times the initial number of nuclei of a radioactive isotope decreases in three years, if in one year it decreases by 4 times.

8.49. Determine which part (%) of the initial number of nuclei of the radioactive isotope will not decay during time  $t$ , which is twice as long as the average lifetime of the radioactive nucleus.

8.50. Determine the half-life of the radioactive isotope if  $5/8$  of the initial number of nuclei of this isotope has decayed in a time of 849 seconds.

8.51. Constant radioactive decay of the isotope  $^{210}_{82}\text{Pb}$  is  $10^{-9} \text{ c}^{-1}$ . Determine the time during which  $2/5$  of the initial number of nuclei of this radioactive isotope will decay.

8.52. The initial mass of the radioactive isotope iodine  $^{131}_{53}\text{I}$  (half-life  $T_{1/2}=8$  days) equals 1 g. Determine: initial activity of the isotope; isotope activity after 3 days.

8.53. The activity of some radioactive isotope at the initial time point was 100 Bq. Determine the activity of this isotope for a time equal to half of the half-life period.

8.54. As a result of the decay of 1 g of radium  $^{226}\text{Ra}$  during 1 hour formed some mass of helium, which occupies under normal conditions volume  $43 \text{ mm}^3$ . Find the constant Avogadro from this data.

8.55. What mass  $m_2$  of radioactive isotope  $^{210}_{83}\text{Bi}$  should be added to the mass of 5 mg of non-radioactive isotope  $^{209}_{83}\text{Bi}$ , so that 10 days after that the ratio of the number of atoms that have decayed to the number of atoms that have not decayed is 50%? The decay constant of the isotope  $^{210}_{83}\text{Bi}$  is  $0,14 \text{ days}^{-1}$ .

8.56. The kinetic energy of the  $\alpha$ -particle escaping from the nucleus of the polonium atom  $^{214}_{84}\text{Po}$  at a radioactive decay of 7,68 MeV. Find: the speed of the  $\alpha$ -particle and the total energy released by the departure of the  $\alpha$ -particle.

8.57. The 1 g isotope  $^{238}\text{U}$  emits  $1,24 \cdot 10^4$   $\alpha$ -particles per second. Determine the half-life of the isotope and the activity of the preparation.

8.58. Find the decay constant  $\lambda$  and the average lifetime  $\tau$  of radioactive  $^{55}\text{Co}$ , if its activity decreases by 4% in 1 hour.

8.59. The intensity of the narrow beam of  $\gamma$ -radiation after passing through a layer of lead with a thickness of 4 cm decreased by 8 times. Determine the energy of  $\gamma$ -photons and thickness  $x_{1/2}$  of the half-absorption layer.

8.60. A narrow beam of  $\gamma$ -radiation (the energy of  $\gamma$ -photons is 2,4 MeV) passes through a concrete slab 1 m thick. What thickness does a slab of cast iron give the same attenuation of this beam of  $\gamma$ -radiation?

8.61. The cast iron plate reduces the intensity of a narrow beam of  $\gamma$ -radiation (the energy of  $\gamma$ -photons is 2,8 MeV) by 10 times. How many times does the intensity of this bundle reduce the lead plate of the same thickness?

8.62. What proportion of the initial number of nuclei will decay in 1 day and in 15 years?

8.63. A preparation containing 1 g of radium-226 is placed in a glass ampoule. What amount of radon-222 will be formed in the ampoule in a time equal to the half-life of radon?

8.64. The initial and final elements of the radioactive family are given:  $^{238}_{92}\text{U} \rightarrow ^{206}_{82}\text{Pb}$ . How many  $\alpha$ - and  $\beta$ -transformations have occurred in the family?

8.65. The initial and final elements of the radioactive family are given:  $^{232}_{90}\text{Th} \rightarrow ^{206}_{82}\text{Pb}$ . How many  $\alpha$ - and  $\beta$ -transformations have occurred in the family?

8.66. How much water can be heated from 0°C to boiling if you use all the heat that was released during the reaction  ${}^7_3\text{Li}(p, \alpha)$ , at complete decomposition of one gram of lithium? Write the nuclear reaction.

8.67. The initial and final elements of the radioactive family are given:  ${}^{235}_{92}\text{U} \rightarrow {}^{207}_{82}\text{Pb}$ . How many  $\alpha$ - and  $\beta$ -transformations have occurred in the family?

8.68. Compare the energy released in the process of fusion  ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He}$  and separation of the uranium nucleus  ${}^{235}_{92}\text{U}$  (each act of separation is accompanied by the release of 200 MeV of energy), if the same masses of nuclear fuel are consumed in both cases, which are equal to 100 g.

8.69. The nuclear power plant consumes 19,2 kg of uranium  ${}^{235}_{92}\text{U}$  per year. Provided that 200 MeV of energy is released with each act of uranium nucleus separation, the efficiency during electricity production is 25%. Determine the electrical capacity of the nuclear power plant.

8.70. The electron and positron, which have the same energy of 0,7 MeV, are transformed into two identical photons during the collision. Determine the wavelength of the corresponding photon.

8.71. When the lithium isotope is bombarded  ${}^6_3\text{Li}$  with deuterons (deuterium  ${}^2_1\text{H}$  nuclei, two  $\alpha$ -particles are formed. At the same time, energy is released 22,3 MeV. Knowing the masses of deuteron and  $\alpha$ -particle, find the mass of the isotope lithium  ${}^6_3\text{Li}$ .

8.72. Find the electric power of a nuclear power plant, which consumes 0,1 kg of uranium-235 per day, if the efficiency of the plant is 16%.

8.73. Fission of one uranium-235 nucleus releases energy of 200 MeV. What part of the resting energy of the uranium-235 nucleus is the released energy?

8.74. The carbon nucleus  ${}^{14}_6\text{C}$  ejected a negatively charged  $\beta$ -particle and an antineutrino. Determine the total energy of  $\beta$ -decay of the nucleus.

8.75. What energy (in kilowatt hours) can be obtained from the separation of a mass of 1 g of uranium  ${}^{235}_{92}\text{U}$ , if the energy of 200 MeV is released at each act of decay?

## REFERENCE TABLES

**Table 1. Basic physical constants**

Speed of light in vacuum	$c = 299792458 \text{ m/s}$
Constant of universal gravitation	$G = 6,67 \cdot 10^{-11} \text{ m}^3 / (\text{kg} \cdot \text{s})$
Magnetic constant	$\mu_0 = 4\pi \cdot 10^{-7} \text{ Hn/m}$
Electric constant	$\epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$
Planck's constant	$h = 6.63 \cdot 10^{-34} \text{ J} \cdot \text{s}$
Electron mass	$m_e = 9,1 \cdot 10^{-31} \text{ kg}$
Proton mass	$m_p = 1,6726 \cdot 10^{-27} \text{ kg}$
Neutron mass	$m_n = 1,6749 \cdot 10^{-27} \text{ kg}$
Elementary charge	$e = 1,6 \cdot 10^{-19} \text{ Cl}$
Electron specific charge	$-e/m = -1,75881962 \cdot 10^{11} \text{ Cl/kg}$
Mass $\alpha$ -particle	$m_\alpha = 6,64 \cdot 10^{-27} \text{ kg}$
Charge $\alpha$ -particle	$+q = +2e = +3,2 \cdot 10^{-19} \text{ Cl}$
Universal gas constant	$R = 8,314510 \text{ J}/(\text{mole} \cdot \text{K})$
Avogadro's constant	$N_A = 6,0221367 \cdot 10^{23} \text{ mole}^{-1}$
Boltzmann's constant	$k = 1,380658 \cdot 10^{-23} \text{ J/K}$
Loschmidt number	$N_L = 2.7 \cdot 10^{25} \text{ m}^{-3}$
Stefan - Boltzmann constant	$\sigma = 5,67051 \cdot 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4)$
Bohr magneton for an electron	$\mu_B = 9,273096 \cdot 10^{-24} \text{ J/Tl}$
Nuclear magneton	$\mu_N = 5,050951 \cdot 10^{-27} \text{ J/Tl}$
Atomic mass unit	$1 \text{ a.m.u.} = 1,66 \cdot 10^{-27} \text{ kg}$

**Table 2. Astronomical quantities**

Average density of the Earth	$5500 \text{ kg/m}^3$
Average Earth radius	$6,37 \cdot 10^6 \text{ m}$
Mass of the Earth	$5,96 \cdot 10^{24} \text{ kg}$
Average density of the Sun	$1400 \text{ kg/m}^3$
Radius of the Sun	$6,95 \cdot 10^8 \text{ m}$
Mass of the Sun	$1,97 \cdot 10^{30} \text{ kg}$

Radius of the Moon	$1,74 \cdot 10^6 m$
Mass of the Moon	$7,3 \cdot 10^{22} kg$
Average distance between the centers of the Moon and Earth	$3,84 \cdot 10^8 m$
Average distance between the centers of Earth and the Sun	$1,5 \cdot 10^{11} m$
Moon rotation period around Earth	$27,3 days = 2,36 \cdot 10^6 s$

**Table 3. Density of substance,  $\rho$ , kg/m<sup>3</sup>**

Gases under normal conditions ( $T_0 = 273,15 K$ , $p_0 = 1,01 \cdot 10^5 Pa$ )			
Hydrogen	0,089	Neon	0,900
Helium	0,178	Carbon dioxide	1,977
Nitrogen	1,250	Methane	0,717
Oxygen	1,429	Air	1,293
Liquids			
Benzene ( $t = 20^0 C$ )	879	Turpentine ( $t = 16^0 C$ )	858
Water ( $t = 4^0 C$ )	1000	Ethyl alcohol ( $t = 0^0 C$ )	789
Kerosene ( $t = 0^0 C$ )	800	Methyl alcohol ( $t = 0^0 C$ )	792
Glycerin ( $t = 0^0 C$ )	1260	Toluene ( $t = 18^0 C$ )	870
Castor oil ( $t = 20^0 C$ )	950	Mercury ( $t = 0^0 C$ )	13596
Solid bodies at 293 K ( $\rho \cdot 10^{-3}$ , kg/m <sup>3</sup> )			
Aluminum	2,69	Cast tin	7,23
Iron, chemically clean	7,86	Cast steel	7,7 – 8,0
Brass	8,3 – 8,7	Lead	11,22–11,44
Ice ( $t = 0^0 C$ )	0,91	Silver	10,42–10,57
Electrolytic copper	8,88 – 8,96	Zinc	6,86 – 7,24
Nickel	8,4 – 9,2	Cast iron	6,6 – 7,3

**Table 4. Coefficient of internal friction of some gases at  $t = 0^0 C$**

Gas	$\eta \cdot 10^6$ , Pa·s
Nitrogen	16,7
Hydrogen	8,4
Carbon dioxide	14,0

Helium	18,9
Oxygen	19,2
Argon	22,9
Dry air*	17,5

**Table 5. Dynamic viscosity coefficient of some liquids at  $t = 20^{\circ} C$**

Liquid	$\eta \cdot 10^3, Pa \cdot s$
Acetone	0,31
Ethylene glycol	16,1
Methyl alcohol	0,544
Benzene	0,673
Water	1,005
Glycerin	1480
Castor oil	970
Mercury	1,590
Ethyl alcohol	1,2

**Table 6. Coefficient of surface tension of some liquids at the "liquid – air" limit at  $t = 20^{\circ} C$**

Liquid	$\alpha \cdot 10^3, N/m$
Aniline	42,9
Acetone	23,7
Water	72,6
Nitric acid (70%)	59,4
Sulfuric acid (85%)	57,4
Acetic acid	27,8
Benzene	30
Glycerin	66
Castor oil	36,4
Kerosene	24
Gasoline	21
Soap solution	40
Milk	46
Oil	30
Mercury	510
Ethyl alcohol	22
Ethyl ether	16,9

**Table 7. Psychrometric table of relative air humidity  $\varphi$ , %**

Indications of dry thermometer		Difference between indications of dry and wet thermometers											
		0	1	2	3	4	5	6	7	8	9	10	11
$K$	$^{\circ}C$	Relative humidity $\varphi$ , %											
273	0	100	82	63	45	28	11						
274	1	100	83	65	48	32	16						
275	2	100	84	68	51	35	20						
276	3	100	84	69	54	39	24	10					
277	4	100	85	70	56	42	28	14					
278	5	100	86	72	58	45	32	19	6				
279	6	100	86	73	60	47	35	23	10				
280	7	100	87	74	61	49	37	26	14				
281	8	100	87	75	63	51	40	28	18	7			
282	9	100	88	76	64	53	42	31	21	11			
283	10	100	88	76	65	54	44	34	24	14	4		
284	11	100	88	77	66	56	46	36	26	17	8		
285	12	100	89	78	68	57	48	38	29	20	11		
286	13	100	89	79	69	59	49	40	31	23	14	6	
287	14	100	90	79	70	60	51	42	33	25	17	9	
288	15	100	90	80	71	61	52	44	36	27	20	12	5
289	16	100	90	81	71	62	54	45	37	30	22	15	8
290	17	100	90	81	72	64	55	47	39	32	24	17	10
291	18	100	91	82	73	64	56	48	41	34	26	20	13
292	19	100	91	82	74	65	58	50	43	35	29	22	15
293	20	100	91	83	74	66	59	51	44	37	30	24	18
294	21	100	91	83	75	67	60	52	46	39	32	26	20
295	22	100	92	83	76	68	61	54	47	40	34	28	22
296	23	100	92	84	76	69	61	55	48	42	36	30	24
297	24	100	92	84	77	69	62	56	49	43	37	31	26
298	25	100	92	84	77	70	63	57	50	44	38	33	27
299	26	100	92	85	78	71	64	58	51	45	40	34	29
300	27	100	92	85	78	71	65	59	52	47	41	36	30
301	28	100	93	85	78	72	65	59	53	48	42	37	32
302	29	100	93	86	79	72	66	60	54	49	43	38	33
303	30	100	93	86	79	73	67	61	55	50	44	39	34

**Table8. Pressure and density of saturated water vapor at different temperatures**

$t, ^\circ\text{C}$	$p_s, \text{GPa}$	$\rho_s, \text{g/m}^3$	$t, ^\circ\text{C}$	$p_s, \text{GPa}$	$\rho_s, \text{g/m}^3$	$t, ^\circ\text{C}$	$p_s, \text{GPa}$	$\rho_s, \text{g/m}^3$
0	6,11	4,84	11	13,12	10,0	22	26,44	19,4
1	6,57	5,22	12	14,03	10,7	23	28,93	20,6
2	7,05	5,60	13	14,97	11,4	24	29,84	21,8
3	7,59	5,98	14	15,99	12,1	25	31,68	23,0
4	8,13	6,40	15	17,05	12,8	26	33,61	24,4
5	8,72	6,48	16	18,17	13,6	27	35,65	25,8
6	9,35	7,30	17	19,37	14,5	28	37,80	27,2
7	10,01	7,80	18	20,64	15,4	29	40,05	28,7
8	10,73	8,30	19	21,97	16,3	30	42,42	30,8
9	11,48	8,80	20	23,65	17,3	31	44,93	32,1
10	12,28	9,40	21	24,87	18,3	32	47,54	33,9

**Table 9. Effective diameters of molecules and atoms  $d, m$**

Nitrogen ( $N_2$ )	$3.1 \cdot 10^{-10}$	Water vapor ( $H_2O$ )	$2.9 \cdot 10^{-10}$
Argon ( $Ar$ )	$2.9 \cdot 10^{-10}$	Helium ( $He$ )	$1.9 \cdot 10^{-10}$
Hydrogen ( $H_2$ )	$2.7 \cdot 10^{-10}$	Oxygen ( $O_2$ )	$2.9 \cdot 10^{-10}$
Carbon dioxide ( $CO_2$ )	$3.3 \cdot 10^{-10}$	Carbon monoxide ( $CO$ )	$3.2 \cdot 10^{-10}$
		Chlorine ( $Cl_2$ )	$3.6 \cdot 10^{-10}$

**Table 10. Elastic properties of solids: Young's modulus  $E, Pa$  ;shear modulus  $\beta, Pa$  ;Poisson's ratio  $\nu$  ;strength limit  $\sigma_s, Pa$**

Material	$E \cdot 10^{-10}$	$\beta \cdot 10^{-10}$	$\nu$	$\sigma_s \cdot 10^{-8}$
Aluminum	6,1 – 7,4	2,2 – 2,6	0,33	0,98 – 3,90
Wrought iron	20 – 22	6,9 – 8,3	0,28	3,90 – 5,90
Steel	20 – 22	7,8 – 8,1	0,28	4,9 – 15,7
Cast irons are gray and white	7,4 – 17,6	4,9	0,23 – 0,27	1,17 – 1,27
Brass	7,8 – 9,8	2,6 – 3,6	0,3 – 0,4	0,98 – 4,90
Copper	10 – 13	3,8 – 4,7	0,31 – 0,40	1,56 – 4,41
Lead	1,5 – 1,7	0,54	0,44	0,0196

**Table 11. Specific heat capacities of gases and the adiabatic indicator**

Substance	$c_p, \text{J/kg} \cdot \text{K}$	$c_v, \text{J/kg} \cdot \text{K}$	$\gamma$
Nitrogen	1051	745	1.41
Ammonia	2244	1675	1.34
Hydrogen	14269	10132	1.41

**Table 12. Dielectric permeabilities of some substances at  $t = 20^\circ \text{C}$** 

Substance	$\epsilon$	Substance	$\epsilon$	Substance	$\epsilon$
Nitrogen	1,00060	Water	81	Paraffin	2
Hydrogen	1,00027	Kerosene	2,1	Plexiglass	3,3
Air	1,00058	Glycerin	39,1	Glass	7
Oxygen	1,00055	Oil	2,5	Mica	6
Carbon dioxide	1,00096	Ethyl alcohol	25	Polychlorvinyl	4
		Turpentine	2,2	Textolite	7
				Polyethylene	2,3
				Kapron	4,2

**Table 13. Specific resistance of some substances at  $t = 20^\circ \text{C}$** 

Substance	$\rho \cdot 10^8, \text{Ohm} \cdot \text{m}$	Substance	$\rho \cdot 10^8, \text{Ohm} \cdot \text{m}$
Aluminum	2,8	Nikelin	42
Tungsten	5,5	Nichrome	110
Graphite	1300	Platinum	10
Ebonite	$10^{18}$	Mercury	95,8
Iron	10	Lead	21
Gold	2,4	Silver	1,6
Constantan	50	Steel	12
Brass	7,1	Porcelain	$10^{21}$
Manganin	45	Fehral	120
Copper	1,7	Chromel	140

**Таблиця 14. Температурний коефіцієнт опору деяких речовин**

Substance	$\alpha, 1/\text{K}$	Substance	$\alpha, 1/\text{K}$
Aluminum	0,0042	Nickel	0,0001
Tungsten	0,0046	Nichrome	0,0001
Coal	0,0008	Tin	0,0042

Iron	0,0060	Platinum	0,0038
Iridium	0,0039	Radium	0,0044
Constantan	0,000003	Mercury	0,0009
Brass	0,0010	Lead	0,0042
Manganite	0,000030	Silver	0,0040
Copper	0,0043	Steel	0,0050
Sodium	0,0044	Zinc	0,0037
Neusilber	0,0003	Cast iron	0,0010

**Таблиця 15. Магнітна сприйнятливість та проникність для деяких матеріалів**

Medium	Susceptibility $\chi$	Permeability $\mu$
Permaloy	8000	7960
Electric steel	4000	3980
Ferrite (nickel-zinc)	–	16-640
Ferrite (manganese-zinc)	–	640
Steel	700	696
Nickel	100	99,5
Platinum	$2,65 \cdot 10^{-4}$	1,0003
Aluminum	$2,22 \cdot 10^{-5}$	1,00002
Hydrogen	$8 \cdot 10^{-9}$	1
Vacuum	0	1
Sapphire	$-2,1 \cdot 10^{-7}$	0,999994
Copper	$-6,4 \cdot 10^{-6}$	0,999994
Water	$-8 \cdot 10^{-6}$	0,999992

**Table 16. Curie point of some substances**

Substance	Curie point, $K$	
	Upper	Lower
<i>Ferroelectrics</i>		258,0
Barium metatitanate	373	
Segnetov salt	295,5	
<i>Ferromagnets</i>		
Iron	1043	
Iron siliceous (4.3% Si)	963	
Cobalt	1403	
Nickel	631	
Permaloy	823	
Permaloy	845	
Heisler alloy	603	

**Table 17. Absolute refractive indicators of some media**

Substance	$n$	Substance	$n$
Diamond	2,42	Sugar	1,56
Aniline	1,59	Carbon disulfide	1,63
Acetone	1,36	Sylvin	1,49
Benzene	1,50	Turpentine	1,51
Water	1,33	Methyl alcohol	1,33
Air	1,0003	Ethyl alcohol	1,36
Glycerin	1,47	Glass (light crown)	1,5
Rock salt	1,54	Glass (light crown)	1,6-1,8
Quartz	1,54	Carbon tetrachloride	1,46
Ice	1,31		

**Table 18. Work of electrons output from some metals**

Metal	$A, eV$	$A, J$
Potassium	2,2	$3,5 \cdot 10^{-19}$
Lithium	2,3	$3,7 \cdot 10^{-19}$
Sodium	2,5	$4 \cdot 10^{-19}$
Platinum	6,3	$1,01 \cdot 10^{-18}$
Silver	4,7	$7,5 \cdot 10^{-19}$
Zinc	4,0	$6,4 \cdot 10^{-19}$
Tungsten	4,53	$7,25 \cdot 10^{-19}$
Cesium	1,2	$1,92 \cdot 10^{-19}$

**Table 19. “Red limit” of the photoeffect for some substances**

Substance	$\lambda_{max}, nm$	Substance	$\lambda_{max}, nm$
Barium	484	Platinum	190
Barium on tungsten	1130	Rubidium	573
Tungsten	272	Silver	261
Germanium	272	Thorium on tungsten	473
Copper oxide	239	Cesium	662

Nickel	249	Cesium on tungsten	909
Barium oxide	1235	Cesium on platinum	895

**Table 20. Mass and rest energy of some particles**

Particles	$m_0$		$E_0$	
	a.m.u.	$10^{-27}$ , kg	MeV	$10^{-10}$ , J
Electron	0,0005486	0,000911	0,511	0,00082
Proton	1,007277	1,67251	938,27	1,503
Neutron	1,008665	1,67482	939,56	1,505
$\alpha$ -particle	4,001507	6,64422	3727,3	5,972
Neutral $\pi$ -meson	0,14526	0,241	135	-
Deuteron	2,01355	3,35	1876	3,00

**Table 21. Mass of some neutral atoms (a. m. u.)**

Element	Sequence number	Isotope	Mass
(Neutron)	0	$n$	1,00867
Hydrogen	1	$^1\text{H}$ $^2\text{H}$ $^3\text{H}$	1,007825 2,014102 3,016049
Helium	2	$^3\text{He}$ $^4\text{He}$	3,01603 4,002604
Lithium	3	$^6\text{Li}$ $^7\text{Li}$	6,01513 7,016005
Beryllium	4	$^7\text{Be}$ $^9\text{Be}$ $^{10}\text{Be}$	7,01693 9,01219 10,01354
Boron	5	$^9\text{B}$ $^{10}\text{B}$ $^{11}\text{B}$	9,01333 10,012935 11,009305
Carbon	6	$^{10}\text{C}$ $^{12}\text{C}$	10,00168 12,00000

		$^{13}\text{C}$	13,00335
		$^{14}\text{C}$	14,00324
Nitrogen	7	$^{13}\text{N}$	13,00574
		$^{14}\text{N}$	14,00307
		$^{15}\text{N}$	15,00011
Oxygen	8	$^{16}\text{O}$	15,99491
		$^{17}\text{O}$	16,99913
		$^{18}\text{O}$	17,99916
Element	Sequence number	Isotope	Mass
Aluminum	13	$^{27}\text{Al}$	29,98146
Lead	82	$^{226}\text{Pb}$	205,97446
Radium	88	$^{226}\text{Ra}$	226,02536
Radon	86	$^{226}\text{Rn}$	222,01753

**Table 22. Half-life periods of some nuclides**

Nuclide	Half-life period, $T$	Nuclide	Half-life period, $T$
$^{192}_{77}\text{Ir}$	74,4 days	$^{45}_{20}\text{Ca}$	153 days
$^{14}_6\text{C}$	5730 years	$^{210}_{82}\text{Pb}$	22,3 years
$^{137}_{55}\text{Cs}$	26,6 years	$^{226}_{88}\text{Ra}$	1600 years
$^{60}_{27}\text{Co}$	5,2 years	$^{204}_{81}\text{Te}$	3,56 years
$^{90}_{38}\text{Sr}$	28,1 years	$^{239}_{94}\text{Pu}$	$24,39 \cdot 10^3$ years
$^{235}_{92}\text{U}$	$7 \cdot 10^6$ years	$^{210}_{84}\text{Po}$	138,4 days
$^{238}_{92}\text{U}$	$4,51 \cdot 10^9$ years	$^{212}_{84}\text{Po}$	$3 \cdot 10^{-7}$ seconds

**Table 23. Prefixes of measurement units**

Name	Designation	Multiplier	Name	Designation	Multiplier
ex	E	$10^{18}$	deci	d	$10^{-1}$
peta	P	$10^{15}$	centi	c	$10^{-2}$
tera	T	$10^{12}$	milli	m	$10^{-3}$
giga	G	$10^9$	micro	mc	$10^{-6}$
mega	M	$10^6$	nano	n	$10^{-9}$
kilo	κ	$10^3$	pico	p	$10^{-12}$
hecto	h	$10^2$	femto	f	$10^{-15}$
deck	d	$10^1$	ato	a	$10^{-18}$

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